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Estimation of Population Mean Utilizing Two Neutrosophic Auxiliary Variables with Imprecise Information

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Abstract

This paper introduces two improved almost unbiased estimator for estimating the finite population mean in neutrosophic settings, incorporating two auxiliary variables to handle indeterminate data more effectively. We have improved the classic ratio and product estimator with new estimators (t_{hN}) and (t_{h1N}), offering enhanced accuracy when applied to uncertain real-life data. Through theoretical derivations and empirical validation using agricultural data (rice yield with climatic variables), we demonstrate that our estimators perform better in term of both accuracy and efficiency. The results show significantly higher Percentage Relative Efficiency (PRE) and lower Mean Squared Error (MSE), highlighting the method's effectiveness for scenarios involving imprecise or indeterminate data. This study develops a framework for better statistical estimation by merging neutrosophic logic with classical sampling methods to handle imprecise data effectively.

Keywords: Neutrosophic statistics, Ratio estimator, Product estimator, Auxiliary variables, Population mean, Percentage relative efficiency.

1 | Introduction

The main objective of sampling theory is to enhance the accuracy of estimating unknown population parameters for a study variable by utilizing auxiliary information. This approach is most effective when there is a strong correlation between the study variable and the auxiliary variable. A number of well-known techniques, including as ratio, product, and regression estimators, are used to estimate population parameters

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using auxiliary information. The ratio method of estimation, first introduced by Cochran [1], is particularly useful when the study variable exhibits a strong positive correlation with the auxiliary variable. When there is a significant negative relationship between the study and auxiliary variables, the product method of estimation is used which was initially used by Murthy [2].

Auxiliary information has been utilized to enhance the accuracy of parameter estimation, as discussed by [3]. When auxiliary information is available, the ratio method of estimation gives better accuracy, but only when the relationship between Y and X forms a straight line passing through the origin. If the regression line has a non-zero intercept, the ratio estimator becomes less accurate. In such cases, a regression-type estimator is a better choice because it takes the intercept into account and provides more precise results. A number of studies are conducted using auxiliary information as [4] introduces about ratio estimators that utilize auxiliary attribute information to estimate the population mean of a study variable. An almost unbiased estimator for population coefficient of variation using auxiliary information was developed by Singh et al. [5].

Several studies have been conducted on sampling with two auxiliary variables in classical statistics. In a study, Sharma and Singh [6] has proposed a new ratio type estimator using auxiliary information on two auxiliary variables based on Simple random sampling without replacement (SRSWOR). In a paper Abu-Dayyeh et al. [7] showed how to extend the two classes of estimators if more than two auxiliary variables are available. Kadilar and Cingi [8] utilizing the estimator of [7], and suggested about an estimator using two auxiliary variables in simple random sampling. Ratio cum product type exponential estimator was constructed by Singh et al. [9]. Singh et al. [10] suggested about the efficiency of dual to ratio-cum-product estimator in sample survey. An almost unbiased ratio and product type estimator in systematic sampling is developed by Singh and Singh [11].

Classical statistical approaches determine the population mean based on clear, unambiguous data values, particularly when auxiliary variables are accessible; however, these methods often struggle with real-world data that contains uncertainty, variability, or incomplete information, leading to less reliable estimates in practical applications. Classical estimators often struggle in practical applications due to their rigidity. In contrast, the rising field of neutrosophic estimators offers greater efficiency in dealing with uncertain or indeterminate data.

The presence of uncertainty or indeterminacy in data leads to the development of neutrosophic statistics. Smarandache [12], developed neutrosophic logic as an extension of fuzzy logic, providing a more robust framework to handle uncertainty, ambiguity, and imprecision. By introducing an additional parameter for indeterminacy, it proves especially valuable when dealing with incomplete or unreliable data. A fuzzy concept refers to an idea that is uncertain or imprecise. The concept of fuzzy logic was introduced by Zadeh [13] and is widely applied in fields like artificial intelligence to manage uncertainty and imprecision effectively. The field of fuzzy statistics has evolved considerably, branching into areas like fuzzy regression analysis, fuzzy probability theory, forecasting using fuzzy time series. The scope also covers confidence interval estimation from imprecise data, operational research applications, hypothesis testing under fuzziness, and challenges with uncertain arrival or service rates. To tackle the oversight of indeterminacy in fuzzy statistics, neutrosophic statistics has emerged as a powerful alternative. It serves as an extension of both fuzzy and classical statistical models and offers a way to quantify the uncertainty present in imprecise data. The concept was originally proposed by Smarandache and has been widely discussed in subsequent literature, particularly in sources [14–18]. The novelty of the neutrosophic framework is further highlighted in the works listed in [19–24].

This study concentrates on neutrosophic logic and statistics, especially in the context of neutrosophic statistics derived from neutrosophic sets or logic. Neutrosophic approaches are employed in situations where fuzzy or intuitionistic statistical techniques are insufficient to capture the indeterminate nature of uncertain or imprecise data. The neutrosophic ratio-type estimators for estimating the population mean introduces by Tahir and Khan [25]. In this article, [25] explores statistical estimation methods under uncertainty using neutrosophic approaches. An almost unbiased estimator for population mean is discussed by [26], using neutrosophic auxiliary information. A neutrosophic estimator that leverages the medians of two auxiliary variables to estimate the finite population mean more accurately is developed by Singh and Tiwari [27]. The estimation of finite population mean in case of neutrosophic ranked set sampling scheme is determines by Singh and Kumari [28].

In this paper, we have constructed two neutrosophic almost unbiased estimator for the estimation of neutrosophic finite population mean using two auxiliary variables under indeterminacy. The first estimator integrates ratio and product approaches with weighting coefficients (α_{0N}, α_{1N} and α_{2N}), to reduces bias and enhance precision. The second estimator combines ratio and exponential term with coefficients (l_{0N}, l_{1N} and l_{2N}). This work extends neutrosophic sampling theory, offering valid solutions for real-world applications with uncertain or imprecise measurements.

2 | Mathematical Notations and Methodology

The neutrosophic observations are represented in form of 'Z_N' which' is expressed as $Z_N = Z_L + Z_U I_N$, where $I_N \in [I_L, I_U]$ and $Z_N \in [Z_L, Z_U]$. Here, 'Z_L' and 'Z_U' represent the lower and upper bounds of the neutrosophic variable 'Z_N'. The term 'I_N' indicates the degree of indeterminacy in 'Z_N', taking values of 0 to 1. This formulation highlights that 'Z_N' is defined in an interval form, meaning any subsequent calculations using 'Z_N' will yield an interval value rather than a single-point result.

Let U_N be a neutrosophic finite population with N_N units $U_{1N}, U_{2N}, U_{3N}, \ldots\ldots.\ldots\ldots\ldots U_{NN}$. This finite population's units are identifiable because each one has a unique label between 1 and N, and each unit's label is known.

Let $y_N \in [y_L, y_U]$ and, $x_N \in [x_L, x_U]$, $z_N \in [z_L, z_U]$ represents the study variate and auxiliary variates respectively with values y_{iN} and (x_{iN}, z_{iN}) on the unit U_{iN} (i = 1, 2, ..., N). Where x_N and y_N have a positive correlation and z_N and y_N have a negative correlation. We want to estimate the finite population mean $\bar{Y}_N = \frac{1}{N_N}\sum_{i=1}^{N} Y_{i_N}$ assuming that the population means of auxiliary variables $\bar{X}_N$ and $\bar{Z}_N$ are known. Assume that a simple random sample of size n_N is drawn without replacement from U_N.

The parameters for the population and sample are given as

$S_{yN}^2 = \frac{1}{(N_N-1)}\sum_{i=1}^{N}(Y_{iN} - \bar{Y}_N)^2$ is the population mean square of the neutrosophic study variable Y_N.

$S_{xN}^2 = \frac{1}{(N_N-1)}\sum_{i=1}^{N}(X_{iN} - \bar{X}_N)^2$ is the population mean square of the neutrosophic auxiliary variable X_N.

$S_{zN}^2 = \frac{1}{(N_N-1)}\sum_{i=1}^{N}(Z_{iN} - \bar{Z}_N)^2$ is the population mean square of the neutrosophic auxiliary variable Z_N.

$S_{xyN} = \frac{1}{(N_N-1)}\sum_{i=1}^{N}(X_{iN} - \bar{X}_N)(Y_{iN} - \bar{Y}_N)$ is the population covariance of the neutrosophic study and auxiliary variable Y_N and X_N.

$S_{yzN} = \frac{1}{(N_N-1)}\sum_{i=1}^{N}(Y_{iN} - \bar{Y}_N)(Z_{iN} - \bar{Z}_N)$ is the population covariance of the neutrosophic study and auxiliary variable Y_N and Z_N.

$S_{xzN} = \frac{1}{(N_N-1)}\sum_{i=1}^{N}(X_{iN} - \bar{X}_N)(Z_{iN} - \bar{Z}_N)$ is the population covariance of the neutrosophic auxiliary variables X_N and Z_N.

$s_{yN}^2 = \frac{1}{(n_N - 1)}\sum_{i=1}^{n}(y_{iN} - \bar{y}_N)^2$ is the sample mean square of the neutrosophic study variable y_N.

$s_{xN}^2 = \frac{1}{(n_N - 1)}\sum_{i=1}^{n}(x_{iN} - \bar{x}_N)^2$ is the sample mean square of the neutrosophic auxiliary variable x_N.

From the above parameters defined a number of existing estimators as given bellow:

The conventional sample mean estimator has been introduced in neutrosophic framework as

$$t_{0N} = \bar{y}_N \text{, where, } t_{0N} \in [t_{0L}, t_{0U}]. \quad \textbf{(1)}$$

The variance of the estimator (t_{0N}) up to the first order approximation is given by

$$V(t_{0N}) = \bar{Y}^2 \gamma_N C_{yN}^2. \quad \textbf{(2)}$$

In neutrosophic framework, the conventional ratio estimators (t_{RN}) for the population mean $(\bar{Y}_N)$ is defined as

$$t_{RN} = \left(\frac{\bar{y}_N}{\bar{x}_N}\right)\bar{X}_N \text{, where, } t_{RN} \in [t_{RNL,}\ t_{RNU}]. \quad \textbf{(3)}$$

Bias and the Mean Square Errors (MSEs) of estimator (t_{RN}) are given by

$$\text{Bias}(t_{RN}) = \bar{Y}_N \gamma_N \left[C_{xN}^2 - \rho_{yxN} C_{yN} C_{xN}\right]. \quad \textbf{(4)}$$

$$\text{MSE}(t_{RN}) = \bar{Y}_N^2 \gamma_N \left[C_{yN}^2 + C_{xN}^2 - 2\rho_{yxN} C_{yN} C_{xN}\right]. \quad \textbf{(5)}$$

Motivated by Murthy [2] Product estimators (t_{PN}) in neutrosophic framework as

$$t_{PN} = \left(\frac{\bar{y}_N}{\bar{Z}_N}\right)\bar{z}_N,\ t_{PN} \in [t_{PNL,}\ t_{PNU}]. \quad \textbf{(6)}$$

Bias and the MSEs of the estimator (t_{PN}) are given by

$$\text{Bias}(t_{PN}) = \bar{Y}_N \gamma_N \rho_{yzN} C_{yN} C_{zN}, \quad \textbf{(7)}$$

$$\text{MSE}(t_{PN}) = \bar{Y}_N^2 \gamma_N \left[C_{yN}^2 + C_{zN}^2 + 2\rho_{yzN} C_{yN} C_{zN}\right], \quad \textbf{(8)}$$

where, $\bar{y}_N = \frac{1}{n_N}\sum_{i=1}^{n} x_{iN}$, $\bar{x}_N = \frac{1}{n_N}\sum_{i=1}^{n} y_{iN}$ and $\bar{z}_N = \frac{1}{n_N}\sum_{i=1}^{n} z_{iN}$ are the sample means of neutrosophic variables y_N, x_N and z_N respectively.

Here, $\gamma_N = \frac{1}{n_N}(1 - f_N)$, $\gamma_N \in [\gamma_{NL,} \gamma_{NU}]$, $f_N = \frac{n_N}{N_N}$, $f_N \in [f_{NL,} f_{NU}]$ is known as sampling fraction. $C_{yN} \in [C_{yNL}, C_{yNU}]$ and C_{xN}, $C_{xN} \in [C_{xNL}, C_{xNU}]$ are the population coefficient of variations of neutrosophic study variable $Y_N \in [Y_{NL}, Y_{NU}]$ and neutrosophic auxiliary variables $X_N \in [X_{NL}, X_{NU}]$. Coefficient of variation are defined as, $C_{yN} = \frac{S_{yN}}{\bar{Y}_N}$, $C_{xN} = \frac{S_{xN}}{\bar{X}_N}$ and $C_{zN} = \frac{S_{zN}}{\bar{Z}_N}$.

ρ_N is the correlation coefficient between X_N and Y_N. Where $S_{xN} \in [S_{xNL}, S_{xNU}]$ and $S_{yN} \in [S_{yNL}, S_{yNU}]$ are the standard deviation of auxiliary variable and study variable respectively. $S_{yxN} \in [S_{yxNL}, S_{yxNU}]$ is the population covariance of the study and auxiliary variable Y_N and X_N.

Other parameter is defined as follows in case of neutrosophic population.

$\rho_{yxN} \in [\rho_{yxL}, \rho_{yxU}]$, $\rho_{yzN} \in [\rho_{yzL}, \rho_{yzU}]$ and $\rho_{xzN} \in [\rho_{xzL}, \rho_{xzU}]$.

Let us define sampling errors for both mean and variance of neutrosophic study and auxiliary variables as

$$e_{0N} = \frac{\bar{y}_N - \bar{Y}_N}{\bar{Y}_N},\ e_{1N} = \frac{\bar{x}_N - \bar{X}_N}{\bar{X}_N},\ e_{2N} = \frac{(\bar{z}_N - \bar{Z}_N)}{\bar{Z}_N},$$

$$\bar{y}_N = \bar{Y}_N(1+e_{0N}),\ \bar{x}_N = \bar{X}_N(1+e_{1N}),\ \bar{z}_N = \bar{Z}_N(1+e_{2N}),$$

$$E(e_{0N}) = E(e_{1N}) = E(e_{2N}) = 0,\ E(e_{0N}^2) = \gamma_N C_{yN}^2,\ E(e_{1N}^2) = \gamma_N C_{xN}^2,\ E(e_{2N}^2) = \gamma_N C_{zN}^2.$$

2.1 | Adapted Existing Estimator

Motivated by [29], we enhanced the ratio and product estimation methods and proposed the 'ratio-cum-product estimator (t_{RPN}) for estimating the finite population mean in neutrosophic study as given

$$t_{RPN} = \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right), \text{where, } t_{RPN} \in [t_{RPL}, t_{RPU}]. \tag{9}$$

$$\text{Bias}(t_{RPN}) = \bar{Y}_N \gamma_N \left[C_{xN}^2 - \rho_{yxN} C_{yN} C_{xN} - \rho_{yzN} C_{yN} C_{zN} + \rho_{xzN} C_{xN} C_{zN}\right]. \tag{10}$$

$$\text{MSE}(t_{RPN}) = \bar{Y}_N^2 \gamma_N \left[C_{yN}^2 + C_{xN}^2 + C_{zN}^2 - 2\rho_{yxN} C_{yN} C_{xN} + 2\rho_{yzN} C_{yN} C_{zN} - 2\rho_{xzN} C_{xN} C_{zN}\right]. \tag{11}$$

Follows the study [30], we have introduced various ratio-cum-product estimators for estimating the finite population mean of study variable ($\bar{Y}_N$) under neutrosophic framework. The estimator (t_{RRN}) represents the ratio-cum-ratio estimator given as

$$t_{RRN} = \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{Z}_N}{\bar{z}_N}\right), \text{where, } t_{RRN} \in [t_{RRL}, t_{RRU}]. \tag{12}$$

Bias and the MSE of the estimator t_{RRN} are defined as

$$\text{Bias}(t_{RRN}) = \bar{Y}_N \gamma_N \left[C_{yN}^2 + C_{zN}^2 - \rho_{yxN} C_{yN} C_{xN} - \rho_{yzN} C_{yN} C_{zN} + \rho_{xzN} C_{xN} C_{zN}\right]. \tag{13}$$

$$\text{MSE}(t_{RRN}) = \bar{Y}_N^2 \gamma_N \left[C_{yN}^2 + C_{xN}^2 + C_{zN}^2 - 2\rho_{yxN} C_{yN} C_{xN} - 2\rho_{yzN} C_{yN} C_{zN} + 2\rho_{xzN} C_{xN} C_{zN}\right]. \tag{14}$$

Moreover, we have introduced the product cum product estimator (t_{PPN}) given as,

$$t_{PPN} = \bar{y}_N \left(\frac{\bar{x}_N}{\bar{X}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right),\ t_{PPN} \in [t_{PPL}, t_{PPU}]. \tag{15}$$

Bias and MSE of the estimator (t_{PPN}) are defined as

$$\text{Bias}(t_{PPN}) = \bar{Y}_N \gamma_N \left[\rho_{yxN} C_{yN} C_{xN} + \rho_{yzN} C_{yN} C_{zN} + \rho_{xzN} C_{xN} C_{zN}\right]. \tag{16}$$

$$\text{MSE}(t_{PPN}) = \bar{Y}_N^2 \gamma_N \left[C_{yN}^2 + C_{xN}^2 + C_{zN}^2 + 2\rho_{yxN} C_{yN} C_{xN} + 2\rho_{yzN} C_{yN} C_{zN} + 2\rho_{xzN} C_{xN} C_{zN}\right]. \tag{17}$$

3 | Proposed Almost Unbiased Estimator–I

Let, $t_{0N} = \bar{y}_N$, $t_{1N} = \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{Z}_N}{\bar{z}_N}\right)$ and $t_{2N} = \bar{y}_N \left(\frac{\bar{x}_N}{\bar{X}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right)$ are the three estimators.

Such that t_{0N}, t_{1N} and $t_{2N} \in L$, where L is the set of all possible estimators for estimating the finite population mean.

By definition, the L is a linear variety [31], [32] if,

$$t_{hN} = \sum_{i=0}^{2} \alpha_{iN} t_{iN} \in L, \; t_{hN} \in [t_{hL}, t_{hU}]. \tag{18}$$

$$t_{hN} = \alpha_{0N} \bar{y}_N + \alpha_{1N} \bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{Z}_N}{\bar{z}_N}\right) + \alpha_{2N} \bar{y}_N \left(\frac{\bar{x}_N}{\bar{X}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right). \tag{19}$$

$$\text{For } \sum_{i=0}^{2} \alpha_{iN} = 1, \; \alpha_{iN} \in R. \tag{20}$$

In this context, $\alpha_{iN} (i = 0,1,2)$ stands for statistical constants, and R denotes the collection of real numbers.

Table 1. Members of the proposed family of estimators (t_{hN}).

α_{0N}	α_{1N}	α_{2N}	**Estimators**
1	0	0	$\bar{y}_N$
0	1	0	$\bar{y}_N \left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{Z}_N}{\bar{z}_N}\right)$
0	0	1	$\bar{y}_N \left(\frac{\bar{x}_N}{\bar{X}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right)$

To determine the bias and Mean Squared Error (MSE) of the estimator (t_{hN}), we express estimator (t_{hN}) in terms of the error component as follows:

$$t_{hN} = \bar{Y}_N \left(1 + e_{0N}\right)\left[\alpha_{0N} + \alpha_{1N}\left(1 + e_{1N}\right)^{-1}\left(1 + e_{2N}\right)^{-1} + \alpha_{1N}\left(1 + e_{1N}\right)\left(1 + e_{2N}\right)\right]. \tag{21}$$

Expanding the right-hand side of the *Eq. (21)* and keeping terms up to the second order of e_N's, we obtain

$$t_{hN} = \bar{Y}_N \begin{bmatrix} 1 + e_{0N} + \left(\alpha_{2N} - \alpha_{1N}\right)e_{1N} + \left(\alpha_{2N} - \alpha_{1N}\right)e_{0N}e_{1N} + \left(\alpha_{2N} - \alpha_{1N}\right)e_{2N} \\ + \left(\alpha_{2N} - \alpha_{1N}\right)e_{0N}e_{2N} + \left(\alpha_{2N} + \alpha_{1N}\right)e_{1N}e_{2N} + \alpha_{1N}\left(e_{1N}^2 + e_{2N}^2\right) \end{bmatrix}. \tag{22}$$

By subtracting $\bar{Y}_N$ and then taking the expectation on both sides, we obtain the bias of the estimator (t_{hN}) up to the first-order approximation as

$$\mathrm{Bias}(t_{hN}) = \overline{Y}_N \gamma_N \begin{bmatrix} \alpha_{1N}\left(C_{xN}^2 + C_{zN}^2\right) + H_N \rho_{yxN} C_{yN} C_{xN} \\ +H_N \rho_{yzN} C_{yN} C_{zN} + (\alpha_{1N} + \alpha_{2N}) \rho_{xzN} C_{xN} C_{zN} \end{bmatrix}, \tag{23}$$

where

$$H_N = (\alpha_{2N} - \alpha_{1N}). \tag{24}$$

From *Eq. (22)*, we have

$$\left(t_{hN} - \overline{Y}\right) \cong \left[e_{0N} + H_N e_{1N} + H_N e_{2N}\right]. \tag{25}$$

Squaring both sides of the *Eq. (25)* and then taking the expectation, we obtain the MSE of the estimator (t_{hN}) up to the first-order approximation as

$$\mathrm{MSE}(t_{hN}) = \overline{Y}_N^2 \gamma_N \begin{bmatrix} C_{yN}^2 + H_N^2 C_{xN}^2 + H_N^2 C_{zN}^2 + \\ 2H_N \rho_{yxN} C_{yN} C_{xN} + 2H_N \rho_{yzN} C_{yN} C_{zN} + 2H_N^2 \rho_{xzN} C_{xN} C_{zN} \end{bmatrix}. \tag{26}$$

Which is minimum when

$$H_N = -\left(\frac{\rho_{yxN} C_{yN} C_{xN} + \rho_{yzN} C_{yN} C_{zN}}{C_{xN}^2 + C_{zN}^2 + 2\rho_{xzN} C_{xN} C_{zN}}\right). \tag{27}$$

After substituting H_N in the *Eq. (26)*, we find the minimum MSE of the estimator (t_{hN}) as

$$\mathrm{Min.MSE}(t_{hN}) = \overline{Y}_N^2 \gamma_N \begin{bmatrix} C_{yN}^2 + H_N^2 C_{xN}^2 + H_N^2 C_{zN}^2 + \\ 2H_N \rho_{yxN} C_{yN} C_{xN} + 2H_N \rho_{yzN} C_{yN} C_{zN} + 2H_N^2 \rho_{xzN} C_{xN} C_{zN} \end{bmatrix}. \tag{28}$$

From *Eq. (24)* and *Eq. (27)*, We have

$$\left(\alpha_{2N} - \alpha_{1N}\right) = H_N = \frac{-(\rho_{yxN} C_{yN} C_{xN} + \rho_{yzN} C_{yN} C_{zN})}{\left(C_{xN}^2 + C_{zN}^2 + 2\rho_{xzN} C_{xN} C_{zN}\right)}. \tag{29}$$

Since we have only two equations for three unknowns, it is not possible to uniquely determine α_{iN}'s $(i = 0,1,2)$. To find the values of α_{iN}'s we introduce a linear restriction as

$$\sum_{i=0}^{2} \alpha_{iN} B(t_{iN}) = 0, \tag{30}$$

$$\alpha_{0N} B(t_{0N}) + \alpha_{1N} B(t_{1N}) + \alpha_{2N} B(t_{2N}) = 0, \tag{31}$$

where, $B(t_{iN})$ denotes the bias of the i^{th} estimator.

The *Eq. (20)*, *Eq. (29)* and *Eq. (31)* can be expressed in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & B(t_{1N}) & B(t_{2N}) \end{bmatrix} \begin{bmatrix} \alpha_{0N} \\ \alpha_{1N} \\ \alpha_{2N} \end{bmatrix} = \begin{bmatrix} 1 \\ H_N \\ 0 \end{bmatrix}. \tag{32}$$

From the system of *Eq. (32)*, we get the unique values of the α_{iN}'s as

$$\alpha_{0N}=\frac{B(t_{1N})+B(t_{2N})+H_N B(t_{2N})-H_N B(t_{1N})}{B(t_{1N})+B(t_{2N})}. \tag{33}$$

$$\alpha_{1N}=-\left(\frac{H_N B(t_{2N})}{B(t_{1N})+B(t_{2N})}\right). \tag{34}$$

$$\alpha_{2N}=\left(\frac{H_N B(t_{1N})}{B(t_{1N})+B(t_{2N})}\right). \tag{35}$$

Such that

$$\alpha_{0N}+\alpha_{1N}+\alpha_{2N}=1. \tag{36}$$

Use of these α_{iN}'s $(i=0,1,2)$ remove the bias up to terms of order $O(n^{-1})$.

4 | Proposed Almost Unbiased Estimator-II

In this section, we introduce another almost unbiased estimator (t_{h1N}) for the finite population mean, utilizing two auxiliary variables under neutrosophic framework. For this purpose, we consider three estimators, m_{0N}, m_{1N}, and m_{2N}, which are defined as

$$m_{0N}=t_{0N}=\bar{y}_N. \tag{37}$$

Bias and the variance of the estimator (m_{0N}) is defined as

$$\text{Bias}(m_{0N})=0. \tag{38}$$

$$\text{Var}(m_{0N})=\bar{Y}_N^2\gamma_N C_N^2. \tag{39}$$

Inspired by [33], the ratio-cum-product estimator within neutrosophic structure, the estimator (t_{RPN}) is given as

$$m_{1N}=t_{RPN}=\bar{y}_N\left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right). \tag{40}$$

The bias and MSE for the ratio-cum-product estimator (m_{1N}) are given by

$$\text{Bias}(m_{1N})=\bar{Y}_N\gamma_N\left[C_{xN}^2-\rho_{yxN}C_{yN}C_{xN}-\rho_{yzN}C_{yN}C_{zN}+\rho_{xzN}C_{xN}C_{zN}\right]. \tag{41}$$

$$\text{MSE}(m_{1N})=\bar{Y}_N^2\gamma_N\left[C_{yN}^2+C_{xN}^2+C_{zN}^2-2\rho_{yxN}C_{yN}C_{xN}+2\rho_{yzN}C_{yN}C_{zN}-2\rho_{xzN}C_{xN}C_{zN}\right]. \tag{42}$$

Following the paper [10], the neutrosophic exponential ratio-cum-product-type estimator $t_{eRPN}=m_{2N}$ has been introduced as

$$m_{2N}=\bar{y}_N\exp\left(\frac{\bar{X}_N-\bar{x}_N}{\bar{X}_N+\bar{x}_N}\right)\exp\left(\frac{\bar{z}_N-\bar{Z}_N}{\bar{z}_N+\bar{Z}_N}\right). \tag{43}$$

The expressions for bias and MSE of the estimator (m_{2N}) are provided as

$$\mathrm{Bias}(m_{2N}) = \bar{Y}_N\gamma_N\left[\frac{3}{8}*C_{xN}^2 - \frac{1}{4}*C_{zN}^2 - \frac{1}{4}\rho_{yxN}C_{yN}C_{xN} + \frac{1}{2}\rho_{yzN}C_{yN}C_{zN} - \frac{1}{4}\rho_{xzN}C_{xN}C_{zN}\right]. \quad \textbf{(44)}$$

$$\mathrm{MSE}(m_{2N}) = \bar{Y}_N^2\gamma_N\left[\begin{array}{l} C_{yN}^2 + \frac{1}{4}C_{xN}^2 + \frac{1}{4}C_{zN}^2 - 2\rho_{yxN}C_{yN}C_{xN} \\ +2\rho_{yzN}C_{yN}C_{zN} - \frac{1}{2}\rho_{xzN}C_{xN}C_{zN} \end{array}\right]. \quad \textbf{(45)}$$

Let $m_{0N} = \bar{y}_N$, $m_{1N} = \bar{y}_N\left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right)$ and $m_{2N} = \bar{y}_N\exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N}\right)\exp\left(\frac{\bar{z}_N - \bar{Z}_N}{\bar{z}_N + \bar{Z}_N}\right)$ are the three estimators expressed. The estimator (t_{h1N}) has been introduced by integrating these estimators.

Where, m_{0N}, m_{1N}, and m_{2N} $\in$ $L(L_{L,}L_U)$ are elements of L, which represents the set of all possible estimators for the finite population mean.

By definition, the L is a linear variety [31], [32] if,

$$t_{h1N} = \sum_{i=0}^{2} l_{iN}m_{iN} \in L(L_L, L_U), \quad \textbf{(46)}$$

$$t_{h1N} = l_{0N}\bar{y}_N + l_{1N}\bar{y}_N\left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right) + l_{2N}\bar{y}_N\exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N}\right)\exp\left(\frac{\bar{z}_N - \bar{Z}_N}{\bar{z}_N + \bar{Z}_N}\right), \quad \textbf{(47)}$$

$$\text{For } \sum_{i=0}^{2} l_{iN} = 1,\ l_{iN} \in R, \quad \textbf{(48)}$$

where $l_{iN}(i = 0,1,2)$ represents statistical constants, and R denotes the set of real numbers.

Table 2. Members of the proposed family of estimators (t_{h1}).

l_{0N}	l_{1N}	l_{2N}	Estimators
1	0	0	$\bar{y}_N$
0	1	0	$\bar{y}_N\left(\frac{\bar{X}_N}{\bar{x}_N}\right)\left(\frac{\bar{z}_N}{\bar{Z}_N}\right)$
0	0	1	$\bar{y}_N\exp\left(\frac{\bar{X}_N - \bar{x}_N}{\bar{X}_N + \bar{x}_N}\right)\exp\left(\frac{\bar{z}_N - \bar{Z}_N}{\bar{z}_N + \bar{Z}_N}\right)$

To determine the bias and MSE of the estimator (t_{h1N}), we express (t_{h1N}) in error terms as

$$t_{h1N} = \bar{Y}_N\left(1+e_{0N}\right)\left[\begin{array}{l} l_{0N} + l_{1N}\left(1+e_{1N}\right)^{-1}\left(1+e_{2N}\right) + \\ l_{2N}\exp\left(-e_{1N}\left(2+e_{1N}\right)^{-1}\right)\exp\left(e_{2N}\left(2+e_{2N}\right)^{-1}\right) \end{array}\right]. \quad \textbf{(49)}$$

Expanding the right-hand side of the *Eq. (49)* and keeping terms up to the second power of e_{0N}'s, we get

$$t_{h1N} = \bar{Y}_N\left[\begin{array}{l} 1 + e_{0N} - \left(\alpha_{1N} + \frac{\alpha_{2N}}{2}\right)e_{1N} + \left(\alpha_{1N} + \frac{\alpha_{2N}}{2}\right)e_{2N} - \left(\alpha_{1N} + \frac{\alpha_{2N}}{2}\right)e_{0N}e_{1N} \\ + \left(\alpha_{1N} + \frac{\alpha_{2N}}{2}\right)e_{0N}e_{2N} - \left(\alpha_{1N} + \frac{1}{4}\alpha_{2N}\right)e_{1N}e_{2N} + \left(\alpha_{1N} + \frac{3}{8}\alpha_{2N}\right)e_{1N}^2 - \frac{1}{4}\alpha_{2N}e_{2N}^2 \end{array}\right]. \quad \textbf{(50)}$$

Subtracting $\bar{Y}_N$ and then taking the expectation on both sides, we obtain the bias of the estimator (t_{h1N}) up to the first-order approximation as

$$\mathrm{Bias}(t_{h1N})=\bar{Y}_N\gamma_N\begin{bmatrix}H_{1N}C_{xN}^2+H_{1N}\rho_{yzN}C_{yN}C_{zN}+\\ \frac{1}{4}l_{2N}\rho_{xzN}C_{xN}C_{zN}-H_{1N}\rho_{yxN}C_{yN}C_{xN}-H_{1N}\rho_{xzN}C_{xN}C_{zN}-\frac{1}{8}l_{2N}C_{xN}^2-\frac{1}{4}l_{2N}C_{zN}^2\end{bmatrix}, \tag{51}$$

or

$$\mathrm{Bias}(t_{h1N})=\bar{Y}_N\gamma_N\left[\begin{Bmatrix}\left(H_{1N}-\frac{1}{8}l_{2N}\right)C_{xN}^2+H_{1N}\rho_{yzN}C_{yN}C_{zN}+\\ \left(\frac{1}{4}l_{2N}-H_{1N}\right)\rho_{xzN}C_{xN}C_{zN}-H_{1N}\rho_{yxN}C_{yN}C_{xN}-\frac{1}{4}l_{2N}C_{zN}^2\end{Bmatrix}\right], \tag{52}$$

where,

$$H_{1N}=(l_{1N}+\frac{1}{2}l_{2N}). \tag{53}$$

From *Eq. (50)*, we have

$$\left(t_{h1N}-\bar{Y}_N\right)\cong\left[e_{0N}+H_{1N}e_{2N}-H_{1N}e_{1N}\right]. \tag{54}$$

Squaring both sides of *Eq. (54)* and then taking the expectation, we obtain the MSE of the estimator (t_{h1N}) up to the first-order approximation as

$$\mathrm{MSE}(t_{h1N})=\bar{Y}_N^2\gamma_N\begin{bmatrix}C_{yN}^2+H_{1N}^2C_{xN}^2+H_{1N}^2C_{zN}^2-2H_{1N}\rho_{yxN}C_{yN}C_{xN}\\ +2H_{1N}\rho_{yzN}C_{yN}C_{zN}-2H_{1N}^2\rho_{xzN}C_{xN}C_{zN}\end{bmatrix}. \tag{55}$$

Which is minimum when

$$H_{1N}=\left(\frac{\rho_{yxN}C_{yN}C_{xN}-\rho_{yzN}C_{yN}C_{zN}}{C_{xN}^2+C_{zN}^2-2\rho_{xzN}C_{xN}C_{zN}}\right). \tag{56}$$

Substituting this value into the *Eq. (55)*, we obtain the minimum MSE of the estimator (t_{h1N}) as

$$\mathrm{Min.MSE}(t_{h1N})=\bar{Y}_N^2\gamma_N\begin{bmatrix}C_{yN}^2+H_{1N}^2C_{xN}^2+H_{1N}^2C_{zN}^2-2H_{1N}\rho_{yxN}C_{yN}C_{xN}\\ +2H_{1N}\rho_{yzN}C_{yN}C_{zN}-2H_{1N}^2\rho_{xzN}C_{xN}C_{zN}\end{bmatrix}. \tag{57}$$

Based on the *Eq. (53)* and *Eq. (56)*, we have

$$H_{1N}=(l_{1N}+\frac{1}{2}l_{2N})=\left(\frac{\rho_{yxN}C_{yN}C_{xN}-\rho_{yzN}C_{yN}C_{zN}}{C_{xN}^2+C_{zN}^2-2\rho_{xzN}C_{xN}C_{zN}}\right). \tag{58}$$

Since we have only two equations for three unknowns, it is not possible to uniquely determine l_{iN}'s $(i=0,1,2)$. To find the values of l_{iN}'s, we introduce a linear restriction as

$$\sum_{i=0}^{2}l_{iN}B(m_{iN})=0, \tag{59}$$

$$l_{0N}B(m_{0N})+l_{1N}B(m_{1N})+l_{2N}B(m_{2N})=0, \tag{60}$$

where $B(m_{iN})$ denotes the bias of the i^{th} estimator.

The *Eq. (48)*, *Eq. (58)* and *Eq. (60)* can be expressed in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & B(m_{1N}) & B(m_{2N}) \end{bmatrix}\begin{bmatrix} l_{0N} \\ l_{1N} \\ l_{2N} \end{bmatrix}=\begin{bmatrix} 1 \\ H_{1N} \\ 0 \end{bmatrix}. \tag{61}$$

Solving the system of *Eq. (61)*, we get the unique values of l_{iN}'s as

$$l_{0N}=\frac{B(m_{2N})-\frac{1}{2}B(m_{1N})-H_{1N}B(m_{2N})+H_{1N}B(m_{1N})}{B(m_{2N})-\frac{1}{2}B(m_{1N})}. \tag{62}$$

$$l_{1N}=\frac{H_{1N}B(m_{2N})}{B(m_{2N})-\frac{1}{2}B(m_{1N})}. \tag{63}$$

$$l_{2N}=\frac{-H_{1N}B(m_{1N})}{B(m_{2N})-\frac{1}{2}B(m_{1N})}. \tag{64}$$

Such that

$$l_{0N}+l_{1N}+l_{2N}=1. \tag{62}$$

Use of these l_{iN}'s $(i=0,1,2)$ remove the bias up to terms of order $O(n^{-1})$.

5 | Empirical Study

In this study, we have considered the dataset from [34], where the neutrosophic study variable (Y_N) corresponds to rice yield, and the neutrosophic auxiliary variables consist of rain sowing (X_{1N}) and rain ripening (X_{2N}) respectively. The dataset is used to analyze the performance of different estimators concerning the neutrosophic study variable $y_N \in [y_{NL}, y_{NU}]$.

Table 3. Descriptive statistics of the given parameter for population 1.

Population 1	
N_N = [9, 9]	C_{YN} = [0.5664, 0.1192]
n_N = [4, 4]	C_{XN} = [0.9258, 0.7679]
f = 138889, 0.138889]	C_{ZN} = [0.7927, 0.8014]
$\bar{Y}_N$ = [3.92222, 18.4333]	ρ_{YXN} = [0.530908, -0.47399]
$\bar{X}_N$ = [11.58889, 35.65556]	ρ_{YZN} = [-0.05753, -0.66775]
$\bar{Z}_N$ = [40.58889, 93.73333]	ρ_{XZN} = [0.428153, 0.73633]

Population 2

In this study, data from Aslam [34] has been considered, where the neutrosophic study variable (Y_N) corresponds to rice yield, and the two neutrosophic auxiliary variables are temperature ripening (X_{1N}) and rain ripening (X_{2N}). This data is used to analyze the efficiency of various estimators for the neutrosophic study variable $y_N \in [y_{NL}, y_{NU}]$.

Table 4. Descriptive statistics of the given parameter for population 2.

Population 2	
N_N = [9, 9]	C_{YN} = [0.5664, 0.1192]
n_N = [4, 4]	C_{XN} = [0.0448, 0.0511]
f = [0.138889, 0.138889]	C_{ZN} = [0.7927, 0.8014]
$\bar{Y}_N$ = [3.92222, 18.4333]	ρ_{YXN} = [-0.2086, 0.2412]
$\bar{X}_N$ = [24.4, 36.6222]	ρ_{YZN} = [-0.0575, -0.6677]
$\bar{Z}_N$ = [40.5889, 93.73333]	ρ_{XZN} = [-0.7674, -0.7445]

Table 5. MSEs and PREs value of existing and proposed estimators for population 1.

Estimator	MSE	I_N	PRE
t_{0N}	[0.6855, 0.6705]	[0, 0.0218]	[100, 100]
t_{RN}	[1.32711, 32.5937]	[0, 0.9593]	[51.6495, 2.0573]
t_{PN}	[3.7064, 24.4037]	[0, 0.8481]	[18.4936, 2.7477]
t_{RRN}	[4.1228, 111.6928]	[0, 0.9631]	[16.6258, 0.6003]
t_{PPN}	[6.2814, 91.4614]	[0, 0.9313]	[10.9125, 0.7331]
t_{RPN}	[1.2166, 14.1130]	[0, 0.9138]	[56.3406, 4.7513]
Min.MSE(t_{hN})	[0.6210, 0.4170]	[0, 0.3284]	[110.3826, 160.7924]
Min.MSE(t_{h1N})	[0.4547, 0.6102]	[0, 0.2548]	[150.7426, 109.8850]

Table 6. MSEs and PREs value of existing and proposed estimators for population 2.

Estimator	MSE	I_N	PRE
t_{0N}	[0.6855, 0.6705]	[0, 0.0217]	[100, 100]
t_{RN}	[0.7124, 0.6551]	[0, 0.0804]	[96.2228, 104.6318]
t_{PN}	[1.9177, 249594]	[0, 0.9232]	[35.7428, 2.7462]
t_{RRN}	[2.0488, 34.1067]	[0, 0.9399]	[33.4558, 2.0097]
t_{PPN}	[1.7829, 22.3436]	[0, 0.9202]	[38.4449, 3.0677]
t_{RPN}	[2.0611, 27.8216]	[0, 0.0.9259]	[33.2566, 2.4637]
Min.MSE(t_{hN})	[0.6819, 0.3567]	[0, 0.4769]	[100.5266, 192.1688]
Min.MSE(t_{h1N})	[0.6820, 0.2979]	[0, 0.5632]	[100.5020, 230.0732]

Table 7. Scalar values shown in this table for this it reduces the bias of the proposed estimator (t_{hN}).

Scalars	Population 1	Population 2
α_{0N}	[1.0336, 0.9721]	[0.9389, 0.8600]
α_{1N}	[0.0429, -0.0111]	[0.0035, 0.0166]
α_{2N}	[-0.0766, 0.0390]	[0.0575, 0.1234]
$H_N = (\alpha_{2N} - \alpha_{1N})$	[-0.1195, 0.0501]	[0.0540, 0.1067]

Table 8. Scalar values shown in this table for this it reduces the bias of the proposed estimator (t_{h1N}).

Scalars	Population 1	Population 2
l_{0N}	[0.2801, 0.9098]	[0.9692, 0.9164]
l_{1N}	[-0.0099, 0.0351]	[0.0292, 0.1013]
l_{2N}	[0.7298, 0.0552]	[0.0016, -0.0177]
$H_{1N} = \left(l_{1N} + \left(\frac{1}{2}\right) l_{2N}\right)$	[0.3550, 0.0627]	[0.0300, 0.0924]

6 | Results and Discussion

The *Table 5* presents a neutrosophic evaluation of various estimators, incorporating MSE, indeterminacy (I_N), and Percentage Relative Efficiency (PRE) as interval-valued metrics to account for uncertainty and variability. The estimators (t_{h1N}) and (t_{hN}) emerge as the top performers, exhibiting low value of MSE ranges [0.4547, 0.6102] and [0.6210, 0.4170], respectively, along with high accuracy. Their high PRE ranges [150.7426, 109.8850] and [110.3826, 160.7924] further emphasize their strong efficiency. Additionally, their moderate indeterminacy ranges [0, 0.2548] and [0, 0.3284] suggest that they remain reliable even under uncertainty, making the better choices for precision-driven applications.

In a similar way, *Table 6* shows that estimators (t_{hN}) and (t_{h1N}) perform the best, with PRE values [100.5266, 192.16188] and [100.5020, 230.0732], it defines about efficiency of the estimators. Their indeterminacy (I_N) values are [0, 0.4769] and [0, 0.5632] suggest that they stay reliable even in uncertain scenarios. The *Table 7* presents the scalars values used in estimators by which it makes linear restriction of the estimators t_{hN}. This value reduces the bias of the proposed estimators. Similarly, the *Table 8* also mention about the constant neutrosophic values in case of proposed estimators (t_{h1N}) make this estimator linear and applying these values reduces the bias of the proposed estimator.

7 | Conclusion

This paper presents an almost unbiased estimator using two auxiliary variables for estimating finite population mean under neutrosophic framework. In this study, we have found that estimator (t_{h1N}) and (t_{hN}) are most efficient estimator comparison than all other existing estimator with low MSE and high PRE values. Using first order of approximation, we have derived Bias and MSE term for the proposed estimators. We have also mentioned the scalars values for both the estimators. It makes the estimator linear and reduces their bias.

For the practical application, we have used agricultural data set. We have found that the neutrosophic estimators is better than classical estimators when data shows indeterminacy. Hence in this case, the proposed neutrosophic estimators are better for the indeterminate data set.

Conflicts of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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