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Soft Symmetric Difference Complement-Union Product of Groups

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Abstract

Soft set theory, recognized for its mathematical precision and algebraic capabilities, provides a robust framework for addressing uncertainty, ambiguity, and variability influenced by parameters. This research introduces a novel binary operation, known as the soft symmetric difference complement-union product, defined for soft sets with parameter domains that exhibit a group-theoretic structure. Based on a solid axiomatic foundation, this operation is demonstrated to satisfy key algebraic properties, including closure, associativity, commutativity, and idempotency, while also being consistent with broader notions of soft equality and subset relationships. It is obtained that the proposed product is a noncommutative semigroup in the collection of soft sets with a fixed parameter set. The study provides an in-depth analysis of the operation's features concerning identity and absorbing elements, as well as its interactions with null and absolute soft sets, all within the framework of group-parameterized domains. The findings suggest that this operation establishes a coherent and structurally robust algebraic system, thereby enhancing the algebraic framework of soft set theory. Furthermore, this research lays the groundwork for the development of a generalized soft group theory, where soft sets indexed by group-based parameters exhibit classical group behaviors through abstract soft operations. The operation's full integration within soft inclusion hierarchies and its compatibility with generalized soft equalities highlight its theoretical importance and broaden its potential applications in formal decision-making and algebraic modeling under uncertainty.

Keywords: Soft sets, Soft subsets, Soft equalities, Soft symmetric difference complement-union.

1 | Introduction

A wide variety of detailed numerical systems have been proposed to represent and analyze frameworks affected by uncertainty, ambiguity, and indeterminacy—essential traits found in areas such as engineering, economics, social sciences, and medical diagnostics. Foundational concepts, such as fuzzy set theory

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introduced by Zadeh [1] and traditional probabilistic models, offer initial tools for tackling these challenges; however, they often encounter epistemological and mathematical constraints. For instance, fuzzy set theory relies on the subjective assignment of membership grades. In contrast, probabilistic approaches require well-defined distributions and repeatable experiments—conditions that are often lacking in real-world, ambiguous scenarios. To overcome these challenges, Molodtsov [2] introduced soft set theory, a parameter-driven and logarithmically adaptable framework that does not rely on probabilistic or fuzzy assumptions. Initial operational definitions by Maji et al. [3] were later refined by Pei and Miao [4] from an information-theoretic perspective, thereby enhancing their relevance in social and multivalued systems. This groundwork was further advanced by Ali et al. [5], who introduced limited and extended operations, greatly enhancing the representational and

mathematical versatility of soft sets. Numerous subsequent studies [6–26] have progressed the field by clarifying definitions, proposing new binary operations, and formalizing generalized soft equivalences.

In recent years, the mathematical framework of soft set theory has undergone significant expansion, as evidenced by various initiatives aimed at developing coherent and extensible parallel operations. These developments have concentrated on generalizing the core concepts of soft subsethood and equivalence. Significant contributions from Qin and Hong [27] laid the foundation for broader theoretical models, which were later refined by Jun and Yang [28] and Liu et al. [29] through the creation of J-soft and L-soft equivalence relations. Feng and Li [30] further advanced the theory by classifying soft subsets under L-equality and demonstrating that specific remainder structures fulfill essential semigroup properties, such as associativity, commutativity, and distributivity. More recent generalizations—such as g-soft, gf-soft, and T-soft equivalences—have been introduced by Abbas et al. [31], [32] and Al-shami [33], and Al-shami and El-Shafei [34], incorporating congruence-based and lattice-theoretic approaches into the mathematical study of soft sets. A breakthrough was achieved by Çağman and Enginoğlu [35], whose foundational modification addressed key inconsistencies and established a coherent and mathematically robust basis for further theoretical developments. This strengthened theoretical foundation has enabled the systematic incorporation of soft set operations into classical mathematical frameworks. The concept of the soft intersection product was first introduced for rings [36], semigroups [37], and groups [38], forming the basis for the development of soft union ring, semigroup, and group theories. Similarly, the soft intersection product was defined for groups [39], semigroups [40], and rings [41], with corresponding algebraic theories subsequently developed. Due to inherent differences among these algebraic structures, the definitions and properties of these products exhibit structural variations. In particular, the presence of a unit element and inverses in groups imparts unique characteristics to the group-based definitions.

Building on this groundwork, the research presents a novel binary operation called the soft symmetric difference complement-union product, which is defined over soft sets with parameter domains influenced by group-theoretic concepts. This operation is formulated within a strictly axiomatic framework and undergoes extensive algebraic examination. Key properties, such as closure, associativity, commutativity, and idempotency, are rigorously proven, along with an investigation into their relationships with identity elements, null and absolute soft sets, and absorbing elements. Additionally, the operation is shown to be entirely compatible with generalized concepts of soft subsethood and soft equality, integrating smoothly with the existing algebraic framework of soft set theory. To assess its theoretical significance and structural role, a comprehensive comparative analysis is conducted against established soft set operations, concentrating on its behavior within layered inclusion hierarchies. It is obtained that the proposed product is a noncommutative semigroup in the collection of soft sets with a fixed parameter set. By abstracting group-theoretic axioms into parameter-dependent soft structures, this operation establishes a foundation for a generalized soft group theory. In this algebraic system, soft sets indexed by group-structured parameters mimic classical group behavior through formally defined operations. Consequently, this work not only introduces a notable algebraic advancement but also creates a solid theoretical basis for applying soft set theory in areas that require formal management of uncertainty, abstract algebraic representation, and multi-criteria decision-making frameworks. Manuscript structure: Section 2 outlines the essential algebraic foundations and formal definitions that support the theoretical framework. Section 3 presents the soft symmetric difference complement-union product and thoroughly explores its related algebraic theory. Finally, Section 4 summarizes the key findings and suggests possible avenues for further development of soft algebra in systems that tackle uncertainty.

2 | Preliminaries

Molodtsov's early work on soft set theory introduced a parameter-based method for modeling uncertainty, but it lacked the required algebraic rigor for integration into abstract algebraic systems. Çağman and Enginoğlu [35] addressed this limitation through an axiomatic reconstruction that provided a solid and logically coherent foundation. The present research relies entirely on this improved framework, which underpins all definitions, operations, and algebraic structures discussed in this study.

Definition 1 ([35]). Let E be a parameter set, U be a universal set, $\mathrm{P(U)}$ be the power set of U, and $\mathcal{H} \subseteq \mathrm{E}$. Then, the soft set $\mathfrak{I}_{\mathcal{H}}$ over U is a function such that $\mathfrak{I}_{\mathcal{H}}\colon \mathrm{E} \to \mathrm{P(U)}$, where for all $\mathrm{w} \notin \mathcal{H}$, $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \emptyset$. That is $\mathfrak{I}_{\mathcal{H}} = \{(\mathrm{w}, \mathfrak{I}_{\mathcal{H}}(\mathrm{w}))\colon \mathrm{w} \in \mathrm{E}\}$.

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2 ([35]). Let $\mathfrak{I}_{\mathcal{H}}$ be an $\mathcal{SS}$. If $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \emptyset$ for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_{\mathrm{E}}$, and if $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \mathrm{U}$, for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is called an absolute SS and indicated by U_{E}.

Definition 3 ([35]). Let $\mathfrak{I}_{\mathcal{H}}$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. If $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) \subseteq \wp_{\aleph}(\mathrm{w})$, for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is a soft subset of $\wp_{\aleph}$ and indicated by $\mathfrak{I}_{\mathcal{H}} \tilde{\subseteq} \wp_{\aleph}$. If $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \wp_{\aleph}(\mathrm{w})$, for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is called soft equal to $\wp_{\aleph}$, and denoted by $\mathfrak{I}_{\mathcal{H}} = \wp_{\aleph}$.

Definition 4 ([35]). Let $\mathrm{f}_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $\mathrm{f}_{\mathcal{H}}$ denoted by ${\mathrm{f}_{\mathcal{H}}}^{\mathrm{c}}$, is defined by the soft set ${\mathrm{f}_{\mathcal{H}}}^{\mathrm{c}}\colon \mathrm{E} \to \mathrm{P(U)}$ such that ${\mathrm{f}_{\mathcal{H}}}^{\mathrm{c}}(\mathrm{e}) = \mathrm{U} \setminus \mathrm{f}_{\mathcal{H}}(\mathrm{e}) = (\mathrm{f}_{\mathcal{H}}(\mathrm{e}))'$, for all $\mathrm{e} \in \mathrm{E}$.

Definition 5 ([42]). Let $\mathfrak{I}_{\mathrm{K}}$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_{\mathrm{K}}$ is called a soft S-subset of $\wp_{\aleph}$, denoted by $\mathfrak{I}_{\mathrm{K}} \tilde{\subseteq}_{\mathrm{S}} \wp_{\aleph}$ if for all $\mathrm{w} \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(\mathrm{w}) = \mathcal{M}$ and $\wp_{\aleph}(\mathrm{w}) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs $\mathfrak{I}_{\mathrm{K}}$ and $\wp_{\aleph}$ are said to be soft S-equal, denoted by $\mathfrak{I}_{\mathrm{K}} =_{\mathrm{S}} \wp_{\aleph}$, if $\mathfrak{I}_{\mathrm{K}} \tilde{\subseteq}_{\mathrm{S}} \wp_{\aleph}$ and $\wp_{\aleph} \tilde{\subseteq}_{\mathrm{S}} \mathfrak{I}_{\mathrm{K}}$.

It is obvious that if $\mathfrak{I}_{\mathrm{K}} =_{\mathrm{S}} \wp_{\aleph}$, then $\mathfrak{I}_{\mathrm{K}}$ and $\wp_{\aleph}$ are the same constant functions, that is, for all $\mathrm{w} \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(\mathrm{w}) = \wp_{\aleph}(\mathrm{w}) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set.

Definition 6 ([42]). Let $\mathfrak{I}_{\mathrm{K}}$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_{\mathrm{K}}$ is called a soft A-subset of $\wp_{\aleph}$, denoted by $\mathfrak{I}_{\mathrm{K}} \tilde{\subseteq}_{\mathrm{A}} \wp_{\aleph}$, if, for each $a, b \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(a) \subseteq \wp_{\aleph}(b)$.

Definition 7 ([42]). Let $\mathfrak{I}_{\mathrm{K}}$ and $\wp_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_{\mathrm{K}}$ is called a soft S-complement of $\wp_{\aleph}$, denoted by $\mathfrak{I}_{\mathrm{K}} =_{\mathrm{S}} (\wp_{\aleph})^{\mathrm{c}}$, if, for all $\mathrm{w} \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(\mathrm{w}) = \mathcal{M}$ and $\wp_{\aleph}(\mathrm{w}) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = \mathrm{U} \setminus \mathcal{D}$.

For additional information on SSs, we refer to [43-59].

3 | Soft Symmetric Difference Complement-Union Product of Groups

This section presents a comprehensive algebraic analysis of the soft symmetric difference complement–union product, a newly introduced binary operation on soft sets derived from group-theoretically organized parameter domains. The study is conducted within a rigorous axiomatic framework, highlighting the essential characteristics of the operation—closure, associativity, commutativity, and idempotency—which confirm its role as an internal operation in soft set algebra. Furthermore, the operation is linked to generalized soft subsethood and soft equality, which are vital for defining morphisms and structuring algebraic substructures. The discussion particularly emphasizes the operation's position within stratified inclusion lattices, ensuring its structural soundness and seamless integration into the broader algebraic framework of soft set theory. It is obtained that the proposed product is a noncommutative semigroup in the collection of soft sets with a fixed parameter set.

From now on, let G be a group, and $\mathrm{S_G(U)}$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $\mathrm{S_G(U)}$ is an $\mathcal{SS}$ parameterized by G. Moreover, let Δ represent the classical symmetric difference operation, and the symmetric difference complement of the family $\mathfrak{B} = \{C_i : i \in I\}$ such that I is an index set, is denoted by

$$\coprod \mathfrak{B} = \coprod_{i \in I} C_i = (C_1 \Delta C_2 \Delta \dots \Delta C_n)'.$$

Definition 8. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s. Then, the soft symmetric difference complement-union product $\mathfrak{I}_G \otimes_{s'/u} \wp_G$ is defined by

$$(\mathfrak{I}_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} \left(\mathfrak{I}_G(y) \cup \wp_G(z)\right), \quad y, z \in G,$$

for all $x \in G$.

Note 1: the soft symmetric difference complement-union product is well-defined in $\mathrm{S_G(U)}$. In fact, let $\mathfrak{I}_G$, $\wp_G$, m_G, $k_G \in \mathrm{S_G(U)}$ such that $(\mathfrak{I}_G, \wp_G) = (m_G, k_G)$. Then, $\mathfrak{I}_G = m_G$ and $\wp_G = k_G$, implying that $\mathfrak{I}_G(x) = m_G(x)$ and $\wp_G(x) = k_G(x)$, for all $x \in G$. Thereby, for all $x \in G$,

$$(\mathfrak{I}_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} \left(\mathfrak{I}_G(y) \cup \wp_G(z)\right) = \coprod_{x=yz} \left(m_G(y) \cup k_G(z)\right) = (m_G \otimes_{s'/u} k_G)(x).$$

Hence,

$$\mathfrak{I}_G \otimes_{s'/u} \wp_G = m_G \otimes_{s'/u} k_G.$$

Example 1. Consider the group $G = \{\sigma, \rho\}$ with the following operation:

$\cdot$	σ	ρ
σ	σ	ρ
ρ	ρ	σ

Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s over $U = D_2 = \{< x, y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$\mathfrak{I}_G = \{(\sigma, \{yx\}), (\rho, \{e, x\})\}$ and $\wp_G = \{(\sigma, \{e, y, yx\}), (\rho, \{e, x\})\}$.

Since $\sigma = \sigma\sigma = \rho\rho$, $(\mathfrak{I}_G \otimes_{s'/u} \wp_G)(\sigma) = \left((\mathfrak{I}_G(\sigma) \cup \wp_G(\sigma))\Delta(\mathfrak{I}_G(\rho) \cup \wp_G(\rho))\right)' = \{e\}$, and since $\rho = \sigma\rho = \rho\sigma$, $(\mathfrak{I}_G \otimes_{s'/u} \wp_G)(\rho) = \left((\mathfrak{I}_G(\sigma) \cup \wp_G(\rho))\Delta(\mathfrak{I}_G(\rho) \cup \wp_G(\sigma))\right)' = \{e, x, yx\}$ is obtained. Hence, $\mathfrak{I}_G \otimes_{s'/u} \wp_G = \{(\sigma, \{e\}), (\rho, \{e, x, yx\})\}$.

Proposition 1. The set $\mathrm{S_G(U)}$ is closed under the soft symmetric difference complement-union product. That is, if $\mathfrak{I}_G$ and $\wp_G$ are two $\mathcal{SS}$s, then so is $\mathfrak{I}_G \otimes_{s'/u} \wp_G$.

Proof: it is obvious that the soft symmetric difference complement-union product is a binary operation in $\mathrm{S_G(U)}$. Thereby, $\mathrm{S_G(U)}$ is closed under the soft symmetric difference complement-union product.

Proposition 2. The soft symmetric difference complement-union product is associative in $\mathrm{S_G(U)}$.

Proof: let $\mathfrak{I}_G$, $\wp_G$, and λ_G be three $\mathcal{SS}$s. Then, for all $x \in G$,

$$\mathfrak{I}_G \otimes_{s'/u} (\wp_G \otimes_{s'/u} \lambda_G)(x) = \coprod_{x=yz} \left(\mathfrak{I}_G(y) \cup (\wp_G \otimes_{s'/u} \lambda_G)(z) \right)$$

$$= \coprod_{x=yz} \left(\mathfrak{I}_G(y) \cup \left(\coprod_{z=mn} (\wp_G(m) \cup \lambda_G(n)) \right) \right)$$

$$= \coprod_{x=y(mn)} \left(\mathfrak{I}_G(y) \cup (\wp_G(m) \cup \lambda_G(n)) \right)$$

$$= \coprod_{x=(ym)n} \left((\mathfrak{I}_G(y) \cup \wp_G(m)) \cup \lambda_G(n) \right)$$

$$= \coprod_{x=an} \left(\left(\coprod_{a=ym} (\mathfrak{I}_G(y) \cup \wp_G(m)) \right) \cup \lambda_G(n) \right)$$

$$= \coprod_{x=an} \left((\mathfrak{I}_G \otimes_{s'/u} \wp_G)(a) \cup \lambda_G(n) \right)$$

$$= (\mathfrak{I}_G \otimes_{s'/u} \wp_G) \otimes_{s'/u} \lambda_G(x).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/u} (\wp_G \otimes_{s'/u} \lambda_G) = (\mathfrak{I}_G \otimes_{s'/u} \wp_G) \otimes_{s'/u} \lambda_G$.

Example 2. Consider the group G and the $\mathcal{SS}$s $\mathfrak{I}_G$ and $\wp_G$ in *Example 1*. Let λ_G be an $\mathcal{SS}$ over $U = \{e, x, y, yx\}$ such that $\lambda_G = \{(\sigma, \{e\}), (\rho, \{x, y\})\}$. Since $\mathfrak{I}_G \otimes_{s'/u} \wp_G = \{(\sigma, \{e\}), (\rho, \{e, x, yx\})\}$, then $(\mathfrak{I}_G \otimes_{s'/u} \wp_G) \otimes_{s'/u} \lambda_G = \{(\sigma, \{e\}), (\rho, \{e, x\})\}$. Moreover, since $\wp_G \otimes_{s'/u} \lambda_G = \{(\sigma, \{e, y\}), (\rho, \{e, x\})\}$, then $\mathfrak{I}_G \otimes_{s'/u} (\wp_G \otimes_{s'/u} \lambda_G) = \{(\sigma, \{e\}), (\rho, \{e, x\})\}$. Thereby, $(\mathfrak{I}_G \otimes_{s'/u} \wp_G) \otimes_{s'/u} \lambda_G = \mathfrak{I}_G \otimes_{s'/u} (\wp_G \otimes_{s'/u} \lambda_G)$.

Proposition 3. The soft symmetric difference complement-union product is not commutative in $S_G(U)$. However, if G is an abelian group, then the soft symmetric difference complement-union product is commutative in $S_G(U)$.

Proof: let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s and G be an abelian group. Then, for all $x \in G$,

$$(\mathfrak{I}_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} (\mathfrak{I}_G(y) \cup \wp_G(z)) = \coprod_{x=zy} (\wp_G(z) \cup \mathfrak{I}_G(y)) = (\wp_G \otimes_{s'/u} \mathfrak{I}_G)(x),$$

implying that $\mathfrak{I}_G \otimes_{s'/u} \wp_G = \wp_G \otimes_{s'/u} \mathfrak{I}_G$.

Proposition 4. The soft symmetric difference complement-union product is not idempotent in $S_G(U)$.

Proof: consider the $\mathcal{SS}$ $\mathfrak{I}_G$ in *Example 1*.

Then, $\mathfrak{I}_G \otimes_{s'/u} \mathfrak{I}_G = \{(\sigma, \{y\}), (\rho, U)\}$, implying that $\mathfrak{I}_G \otimes_{s'/u} \mathfrak{I}_G \neq \mathfrak{I}_G$.

Theorem 1. $(S_G(U), \otimes_{s'/u})$ is a noncommutative semigroup. If G is abelian, then $(S_G(U), \otimes_{s'/u})$ is a commutative semigroup.

Proof: the proof is followed by *Propositions 1-4*.

Proposition 5. Let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$. Then,

I. $\mathfrak{I}_G \otimes_{s'/u} \mathfrak{I}_G = \mathfrak{I}_G{}^c$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/u} \mathfrak{I}_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{I}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathfrak{I}_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$(\mathfrak{J}_G \otimes_{s'/u} \mathfrak{J}_G)(x) = \coprod_{x=yz} (\mathfrak{J}_G(y) \cup \mathfrak{J}_G(z)) = \underbrace{(A \Delta A \Delta \ldots \Delta A)}_{r \text{ times } A, \text{where } r \text{ is even}}{}' = A' = \mathfrak{J}_G{}^c(x).$$

Thereby, $\mathfrak{J}_G \otimes_{s'/u} \mathfrak{J}_G = \mathfrak{J}_G{}^c$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$(\mathfrak{J}_G \otimes_{s'/u} \mathfrak{J}_G)(x) = \coprod_{x=yz} (\mathfrak{J}_G(y) \cup \mathfrak{J}_G(z)) = \underbrace{(A \Delta A \Delta \ldots \Delta A)}_{r \text{ times } A, \text{where } r \text{ is even}}{}' = \emptyset' = U_G(x).$$

Thereby, $\mathfrak{J}_G \otimes_{s'/u} \mathfrak{J}_G = U_G$.

Remark 1. Let $S_G{}^*(U)$ be the collection of all constant $\mathcal{SS}$s. Then, the soft symmetric difference complement-union product is not idempotent in $S_G{}^*(U)$ either.

Proposition 6. Let $\mathfrak{J}_G$ be an $\mathcal{SS}$. Then,

I. $\mathfrak{J}_G \otimes_{s'/u} U_G = U_G \otimes_{s'/u} \mathfrak{J}_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\mathfrak{J}_G \otimes_{s'/u} U_G = U_G \otimes_{s'/u} \mathfrak{J}_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{J}_G$ be an $\mathcal{SS}$.

I. Let $x \in G$ and $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$(\mathfrak{J}_G \otimes_{s'/u} U_G)(x) = \coprod_{x=yz} (\mathfrak{J}_G(y) \cup U_G(z)) = \coprod_{x=yz} (\mathfrak{J}_G(y) \cup U) = \emptyset_G(x).$$

Thus, $\mathfrak{J}_G \otimes_{s'/u} U_G = \emptyset_G$. Similarly, for all $x \in G$,

$$(U_G \otimes_{s'/u} \mathfrak{J}_G)(x) = \coprod_{x=yz} (U_G(y) \cup \mathfrak{J}_G(z)) = \coprod_{x=yz} (U \cup \mathfrak{J}_G(z)) = \emptyset_G(x).$$

Thus, $U_G \otimes_{s'/u} \mathfrak{J}_G = \emptyset_G$.

II. Let $x \in G$ and $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$(\mathfrak{J}_G \otimes_{s'/u} U_G)(x) = \coprod_{x=yz} (\mathfrak{J}_G(y) \cup U_G(z)) = \coprod_{x=yz} (\mathfrak{J}_G(y) \cup U) = U_G(x).$$

Thus, $\mathfrak{J}_G \otimes_{s'/u} U_G = U_G$. Similarly, for all $x \in G$,

$$(U_G \otimes_{s'/u} \mathfrak{J}_G)(x) = \coprod_{x=yz} (U_G(y) \cup \mathfrak{J}_G(z)) = \coprod_{x=yz} (U \cup \mathfrak{J}_G(z)) = U_G(x).$$

Thus, $U_G \otimes_{s'/u} \mathfrak{J}_G = U_G$.

Remark 2. U_G is the absorbing element of the soft symmetric difference complement-union product in $S_G(U)$, where $|G| = r$ and r is a positive even integer.

Proposition 7. Let $\mathfrak{J}_G$ be a constant $\mathcal{SS}$. Then,

I. $\emptyset_G \otimes_{s'/u} \mathfrak{J}_G = \mathfrak{J}_G \otimes_{s'/u} \emptyset_G = \mathfrak{J}_G{}^c$, where $|G| = r$ and r is a positive odd integer.

II. $\emptyset_G \otimes_{s'/u} \mathfrak{J}_G = \mathfrak{J}_G \otimes_{s'/u} \emptyset_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\mathfrak{J}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathfrak{J}_G(x) = A$, where A is a fixed set.

I. Suppose that $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$(\emptyset_G \otimes_{s'/u} \Im_G)(x) = \coprod_{x=yz} (\emptyset_G(y) \cup \Im_G(z)) = \coprod_{x=yz} (\emptyset \cup \Im_G(z)) = \Im_G{}^c(x).$$

Thereby, $\emptyset_G \otimes_{s'/u} \Im_G = \Im_G{}^c$. Similarly, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \emptyset_G)(x) = \coprod_{x=yz} (\Im_G(y) \cup \emptyset_G(z)) = \coprod_{x=yz} (\Im_G(y) \cup \emptyset) = \Im_G{}^c(x).$$

Thereby, $\Im_G \otimes_{s'/u} \emptyset_G = \Im_G{}^c$.

II. Suppose that $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$(\emptyset_G \otimes_{s'/u} \Im_G)(x) = \coprod_{x=yz} (\emptyset_G(y) \cup \Im_G(z)) = \coprod_{x=yz} (\emptyset \cup \Im_G(z)) = U_G(x).$$

Thereby, $\emptyset_G \otimes_{s'/u} \Im_G = U_G$. Similarly, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \emptyset_G)(x) = \coprod_{x=yz} (\Im_G(y) \cup \emptyset_G(z)) = \coprod_{x=yz} (\Im_G(y) \cup \emptyset) = U_G(x).$$

Thereby, $\Im_G \otimes_{s'/u} \emptyset_G = U_G$.

Proposition 8. Let $\Im_G$ be a constant $\mathcal{SS}$. Then,

I. $\Im_G{}^c \otimes_{s'/u} \Im_G = \Im_G \otimes_{s'/u} \Im_G{}^c = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\Im_G{}^c \otimes_{s'/u} \Im_G = \Im_G \otimes_{s'/u} \Im_G{}^c = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\Im_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\Im_G(x) = A$, where A is a fixed set.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$(\Im_G{}^c \otimes_{s'/u} \Im_G)(x) = \coprod_{x=yz} (\Im_G{}^c(y) \cup \Im_G(z)) = \emptyset_G(x).$$

Thereby, $\Im_G{}^c \otimes_{s'/u} \Im_G = \emptyset_G$. Similarly, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \Im_G{}^c)(x) = \coprod_{x=yz} (\Im_G(y) \cup \Im_G{}^c(z)) = \emptyset_G(x).$$

Thereby, $\Im_G \otimes_{s'/u} \Im_G{}^c = \emptyset_G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$(\Im_G{}^c \otimes_{s'/u} \Im_G)(x) = \coprod_{x=yz} (\Im_G{}^c(y) \cup \Im_G(z)) = U_G(x).$$

Thereby, $\Im_G{}^c \otimes_{s'/u} \Im_G = U_G$. Similarly, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \Im_G{}^c)(x) = \coprod_{x=yz} (\Im_G(y) \cup \Im_G{}^c(z)) = U_G(x).$$

Thereby, $\Im_G \otimes_{s'/u} \Im_G{}^c = U_G$.

Proposition 9. Let $\Im_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $\Im_G \widetilde{\subseteq}_S \wp_G$. Then,

I. $\Im_G \otimes_{s'/u} \wp_G = \wp_G \otimes_{s'/u} \Im_G = \wp_G{}^c$, where $|G| = r$ and r is a positive odd integer.

II. $\Im_G \otimes_{s'/u} \wp_G = \wp_G \otimes_{s'/u} \Im_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\Im_G$ and $\wp_G$ be two $\mathcal{SS}$s and $\Im_G \widetilde{\subseteq}_S \wp_G$. Then, for all $x \in G$, $\Im_G(x) = A$, $\wp_G(x) = B$, where A and B are two fixed sets and $A \subseteq B$.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} (\Im_G(y) \cup \wp_G(z)) = \wp_G{}^c(x).$$

Thereby, $\Im_G \otimes_{s'/u} \wp_G = \wp_G{}^c$. Similarly, for all $x \in G$,

$$(\wp_G \otimes_{s'/u} \Im_G)(x) = \coprod_{x=yz} (\wp_G(y) \cup \Im_G(z)) = \wp_G{}^c(x).$$

Thereby, $\wp_G \otimes_{s'/u} \Im_G = \wp_G{}^c$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} (\Im_G(y) \cup \wp_G(z)) = U_G(x).$$

Thereby, $\Im_G \otimes_{s'/u} \wp_G = U_G$. Similarly, for all $x \in G$,

$$(\wp_G \otimes_{s'/u} \Im_G)(x) = \coprod_{x=yz} (\wp_G(y) \cup \Im_G(z)) = U_G(x).$$

Thereby, $\wp_G \otimes_{s'/u} \Im_G = U_G$.

Proposition 10. Let $\Im_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $\Im_G{}^c \widetilde{\subseteq}_A \wp_G$. Then,

I. $\Im_G \otimes_{s'/u} \wp_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\Im_G \otimes_{s'/u} \wp_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: let $\Im_G$ and $\wp_G$ be two $\mathcal{SS}$s and $\Im_G{}^c \widetilde{\subseteq}_A \wp_G$. Then, $\Im_G{}^c(y) \subseteq \wp_G(z)$, for each $y, z \in G$.

I. Let $|G| = r$, where r is a positive odd integer. Then, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} (\Im_G(y) \cup \wp_G(z)) = \emptyset_G(x).$$

Thereby, $\Im_G \otimes_{s'/u} \wp_G = \emptyset_G$.

II. Let $|G| = r$, where r is a positive even integer. Then, for all $x \in G$,

$$(\Im_G \otimes_{s'/u} \wp_G)(x) = \coprod_{x=yz} (\Im_G(y) \cup \wp_G(z)) = U_G(x).$$

Thereby, $\Im_G \otimes_{s'/u} \wp_G = U_G$. Here, note that if $A' \subseteq B$, then $A' \cap B' = (A \cup B)' = \emptyset$.

Remark 3. Let $\Im_G$ and $\wp_G$ be two $\mathcal{SS}$s such that $\Im_G{}^c \widetilde{\subseteq}_S \wp_G$. Then,

I. $\Im_G \otimes_{s'/u} \wp_G = \emptyset_G$, where $|G| = r$ and r is a positive odd integer.

II. $\Im_G \otimes_{s'/u} \wp_G = U_G$, where $|G| = r$ and r is a positive even integer.

Proof: the proof is similar to the proof of *Proposition 10*.

Proposition 11. Let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s. Then, $(\mathfrak{I}_G \otimes_{s'/u} \wp_G)^c = \mathfrak{I}_G \otimes_{s/u} \wp_G$.

Proof: let $\mathfrak{I}_G$ and $\wp_G$ be two $\mathcal{SS}$s. Then, for all $x \in G$,

$$(\mathfrak{I}_G \otimes_{s'/u} \wp_G)^c(x) = \left(\coprod_{x=yz} (\mathfrak{I}_G(y) \cup \wp_G(z))\right)' = \mathop{\triangle}_{x=yz} (\mathfrak{I}_G(y) \cup \wp_G(z))'$$
$$= \mathop{\triangle}_{x=yz} (\mathfrak{I}_G(y) \cup \wp_G(z)) = (\mathfrak{I}_G \otimes_{s/u} \wp_G)(x).$$

Thereby, $(\mathfrak{I}_G \otimes_{s'/u} \wp_G)^c = \mathfrak{I}_G \otimes_{s/u} \wp_G$. Here note that, in classical set theory, $A'\Delta B' = A\Delta B$, where A and B are fixed sets.

4 | Conclusion

This research introduces the soft symmetric difference complement-union product, a novel binary operation on soft sets that is influenced by group-theoretic structures. The operation is examined within a comprehensive algebraic framework, emphasizing its connection to generalized soft equality and its role across various levels of soft subsethood. A comparative analysis highlights the operation's expressive potential and algebraic consistency in relation to existing soft set operations. The study also explores its relationship with key concepts such as null and absolute soft sets, as well as its coherence within group-parameterized binary operations. Key algebraic properties—like closure, associativity, commutativity, and idempotency—are thoroughly validated, including conditions concerning identity, inverse, and absorbing elements. It is obtained that the proposed product is a noncommutative semigroup in the collection of soft sets with a fixed parameter set. The resulting structure demonstrates strong internal consistency, extending traditional algebraic concepts into the domain of soft sets. Ultimately, this operation lays the groundwork for a generalized soft group theory, where soft sets indexed by group-structured parameters exhibit group-like behavior through abstract soft operations. As a result, this work not only strengthens the theoretical basis of soft set theory but also broadens its potential applications in areas such as algebraic modeling and decision-making under uncertainty.

Author Contributions

Zeynep Ay: Investigation, Visualization, Conceptualization, Writing-Review, Validation.

Aslıhan Sezgin: Supervision, Visualization, Conceptualization, Validation, Review.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4-5), 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

[3] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & mathematics with applications*, *45*(4-5), 555-562. https://doi.org/10.1016/S0898-1221(03)00016-6

[4] Pei, D., & Miao, D. (2005). From soft sets to information systems. In *2005 IEEE international conference on granular computing* (Vol. 2, pp. 617-621). IEEE. https://doi.org/10.1109/GRC.2005.1547365

[5] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & mathematics with applications*, *57*(9), 1547-1553. https://doi.org/10.1016/j.camwa.2008.11.009

[6] Yang, C. F. (2008). A note on “soft set theory”[comput. math. appl. 45 (4–5)(2003) 555–562]. *Computers & mathematics with applications*, *56*(7), 1899-1900. https://doi.org/10.1016/j.camwa.2008.03.019

[7] Feng, F., Li, C., Davvaz, B., & Ali, M. I. (2010). Soft sets combined with fuzzy sets and rough sets: A tentative approach. *Soft computing*, *14*(9), 899–911. https://doi.org/10.1007/s00500-009-0465-6

[8] Singh, D., & Onyeozili, I. A. (2012). Notes on soft matrices operations. *ARPN journal of science and technology*, *2*(9), 861–869. http://www.ejournalofscience.org

[9] Zhu, P., & Wen, Q. (2013). Operations on soft sets revisited. *Journal of applied mathematics*, *2013*(1), 105752. https://doi.org/10.1155/2013/105752

[10] Sen, J. (2014). On algebraic structure of soft sets. *Annals of fuzzy mathematics and informatics*, *7*(6), 1013–1020. https://www.researchgate.net/profile/Jayanta-Sen/publication/317004556_On_algebraic_structure_of_soft_sets/links/5922e2ea458515e3d408d838/On-algebraic-structure-of-soft-sets.pdf

[11] Sezgin, A., & Atagün, A. O. (2011). On operations of soft sets. *Computers & mathematics with applications*, *61*(5), 1457–1467. https://doi.org/10.1016/j.camwa.2011.01.018

[12] Stojanović, N. (2021). A new operation on soft sets:extended symmetric difference of soft sets. *Vojnotehnički glasnik/military technical courier*, *69*(4), 779–791. https://doi.org/10.5937/vojtehg69-33655

[13] Sezgin, A., Çağman, N., Atagün, A. O., & Aybek, F. N. (2023). Complemental binary operations of sets and their application to group theory. *Matrix science mathematic*, *7*(2), 114–121. https://doi.org/10.26480/msmk.02.2023.114.121

[14] Sezgin, A., & Dagtoros, K. (2023). Complementary soft binary piecewise symmetric difference operation: A novel soft set operation. *Scientific journal of mehmet akif ersoy university*, *6*(2), 31–45. https://dergipark.org.tr/en/pub/sjmakeu/issue/82332/1365021

[15] Sezgin, A., & Çalişici, H. (2024). A comprehensive study on soft binary piecewise difference operation. *Eskişehir teknik üniversitesi bilim ve teknoloji dergisi b - teorik bilimler*, *12*(1), 32–54. https://doi.org/10.20290/estubtdb.1356881

[16] Sezgin, A., & Yavuz, E. (2024). Soft binary piecewise plus operation: A new type of operation for soft sets. *Uncertainty discourse and applications*, *1*(1), 79–100. https://doi.org/10.48313/uda.v1i1.26

[17] Sezgin, A., & Şenyiğit, E. (2025). A new product for soft sets with its decision-making: Soft star-product. *Big data and computing visions*, *5*(1), 52–73. https://doi.org/10.22105/bdcv.2024.492834.1221

[18] Jiang, Y., Tang, Y., Chen, Q., Wang, J., & Tang, S. (2010). Extending soft sets with description logics. *Computers & mathematics with applications*, *59*(6), 2087–2096. https://doi.org/10.1016/j.camwa.2009.12.014

[19] Sezgin, A., & Demirci, A. M. (2023). A new soft set operation: complementary soft binary piecewise star (*) operation. *Ikonion journal of mathematics*, *5*(2), 24–52. https://doi.org/10.54286/ikjm.1304566

[20] Ali, M. I., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers and mathematics with applications*, *61*(9), 2647–2654. https://doi.org/10.1016/j.camwa.2011.03.011

[21] Neog, T. J., & Sut, D. K. (2011). A new approach to the theory of soft sets. *International journal of computer applications*, *32*(2), 1–6. https://d1wqtxts1xzle7.cloudfront.net/53823653/2011-new_approach_soft_set-libre.pdf?1499755677=&response-content-disposition=inline%3B+filename%3DA_New_Approach_to_the_Theory_of_Soft_Set.pdf&Expires=1740299428&Signature=abUw8rHu4pPoWo9GGYn8lFITB64pDE61N9d

[22] Li, F. (2011). Notes on the soft operations. *ARPN journal of systems and software*, *1*(6), 205–208. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=5e028fe2a1df00303f8012ac465fa114611788d5

[23] Ge, X., & Yang, S. (2011). Investigations on some operations of soft sets. *World academy of science, engineering and technology*, *51*, 1112–1115. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=0f185b915f3223c39847c632a2ea34d1e64a0083

[24] Singh, D., & Onyeozili, I. A. (2012). Some conceptual misunderstandings of the fundamentals of soft set theory 1 1. *ARPN journal of systems and software*, *2*(9), 251–254. https://faculty.alfaisal.edu/gksing/publications/some-conceptual-misunderstandings-of-the-fundamentals-of-soft-set-theory

[25] Professor D. Singh, P. D. S. (2012). Some results on distributive and absorption properties of soft operations. *IOSR journal of mathematics*, *4*(2), 18–30. https://doi.org/10.9790/5728-0421830

[26] Singh, D., & A. Onyeozili, I. (2012). On some new properties of soft set operations. *International journal of computer applications*, *59*(4), 39–44. https://doi.org/10.5120/9538-3975

[27] Qin, K., & Hong, Z. (2010). On soft equality. *Journal of computational and applied mathematics*, *234*(5), 1347–1355. https://doi.org/10.1016/j.cam.2010.02.028

[28] Jun, Y. B., & Yang, X. (2011). A note on the paper "combination of interval-valued fuzzy set and soft set" [Comput. Math. Appl. 58 (2009) 521527]. *Computers and mathematics with applications*, *61*(5), 1468–1470. https://doi.org/10.1016/j.camwa.2010.12.077

[29] Liu, X., Feng, F., & Jun, Y. B. (2012). A note on generalized soft equal relations. *Computers and mathematics with applications*, *64*(4), 572–578. https://doi.org/10.1016/j.camwa.2011.12.052

[30] Feng, F., & Li, Y. (2013). Soft subsets and soft product operations. *Information sciences*, *232*, 44–57. https://doi.org/10.1016/j.ins.2013.01.001

[31] Abbas, M., Ali, B., & Romaguera, S. (2014). On generalized soft equality and soft lattice structure. *Filomat*, *28*(6), 1191–1203. https://doi.org/10.2298/FIL1406191A

[32] Abbas, M., Ali, M. I., & Romaguera, S. (2017). Generalized operations in soft set theory via relaxed conditions on parameters. *Filomat*, *31*(19), 5955–5964. https://doi.org/10.2298/FIL1719955A

[33] Al-Shami, T. M. (2019). Investigation and corrigendum to some results related to g-soft equality and g f-soft equality relations. *Filomat*, *33*(11), 3375–3383. https://www.jstor.org/stable/27382788

[34] Alshami, T., & El-Shafei, M. (2020). $ T $-soft equality relation. *Turkish journal of mathematics*, *44*(4), 1427–1441. https://doi.org/10.3906/mat-2005-117

[35] Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni--int decision making. *European journal of operational research*, *207*(2), 848–855. https://doi.org/10.1016/j.ejor.2010.05.004

[36] Sezgin Sezer, A. (2012). A new view to ring theory via soft union rings, ideals and bi-ideals. *Knowledge-based systems*, *36*, 300–314. https://doi.org/10.1016/j.knosys.2012.04.031

[37] Sezgin, A. (2016). A new approach to semigroup theory I: Soft union semigroups, ideals and bi-ideals. *Algebra letters*, *2016*(3), 1–46. https://scik.org/index.php/abl/article/view/2989

[38] Mustuoglu, E., Sezgin, A., & Kaya, Z. (2016). Some characterizations on soft uni-groups and normal soft uni-groups. *International journal of computer applications*, *155*(10), 1–8. https://doi.org/10.5120/ijca2016912412

[39] Kaygisiz, K. (2012). On soft int-groups. *Annals of fuzzy mathematics and informatics*, *4*(2), 365–375. http://www.afmi.or.kr@fmihttp//www.kyungmoon.com

[40] Sezer, A. S., Agman, N., Atagün, A. O., Ali, M. I., & Turkmen, E. (2015). Soft intersection semigroups, ideals and bi-ideals; a new application on semigroup theory I. *Filomat*, *29*(5), 917–946. https://doi.org/10.2298/FIL1505917S

[41] Sezgin, A., Çağman, N., & Atagün, A. O. (2017). A completely new view to soft intersection rings via soft uni-int product. *Applied soft computing journal*, *54*, 366–392. https://doi.org/10.1016/j.asoc.2016.10.004

[42] Sezgin, A., Durak, İ., & Ay, Z. (2025). Some new classifications of soft subsets and soft equalities with soft symmetric difference-difference product of groups. *Amesia*, *6*(1), 16–32. https://doi.org/10.54559/amesia.1730014

[43] Khan, A., Izhar, M., & Sezgin, A. (2017). Characterizations of abel grassmann's groupoids by the properties of their double-framed soft ideals. *International journal of analysis and applications*, *15*(1), 62–74. http://www.etamaths.com

[44] Atagün, A. O., & Sezer, A. S. (2015). Soft sets, soft semimodules and soft substructures of semimodules. *Mathematical sciences letters*, *4*(3), 235–242. https://www.researchgate.net/profile/Aslihan-Sezgin-2/publication/308938491_Soft_sets_soft_semi-modules_and_soft_substructures_of_semi-modules/links/57fc913f08ae329c3d498c89/Soft-sets-soft-semi-modules-and-soft-substructures-of-semi-modules.pdf

[45] Sezer, A. S., Atagün, A. O., & Cagman, N. (2014). N-group SI-action and its applications to N-group theory. *Fasciculi mathematici*, *54*, 139–153. https://www.researchgate.net/profile/Aslihan-Sezgin-2/publication/263651539_N-group_SI-action_and_its_application_to_N-

group_theory/links/54353a080cf2bf1f1f283279/N-group-SI-action-and-its-application-to-N-group-theory.pdf

[46] Atagün, A. O., & Sezgin, A. (2017). Int-soft substructures of groups and semirings with applications. *Applied mathematics and information sciences*, *11*(1), 105–113. https://doi.org/10.18576/amis/110113

[47] Gulistan, M., Feng, F., Khan, M., & Sezgin, A. (2018). Characterizations of right weakly regular semigroups in terms of generalized cubic soft sets. *Mathematics*, *6*(12), 293. https://doi.org/10.3390/math6120293

[48] Sezgin Sezer, A., Atagün, A. O., & Çağman, N. (2013). A new view to n-group theory: Soft N-groups. *Fasciculi mathematici*, *51*(51), 123–140. https://www.researchgate.net/profile/Aslihan-Sezgin-2/publication/263651532_A_new_view_to_N-group_theory-Soft_N-groups/links/0046353b68f17da045000000/A-new-view-to-N-group-theory-Soft-N-groups.pdf

[49] Jana, C., Pal, M., Karaaslan, F., & Sezgin, A. (2019). (α, β)-soft intersectional rings and ideals with their applications. *New mathematics and natural computation*, *15*(2), 333–350. https://doi.org/10.1142/S1793005719500182

[50] Atagun, A. O., Kamaci, H., Tastekin, I., & Sezgin, A. (2019). P-properties in Near-rings. *Journal of mathematical and fundamental sciences*, *51*(2), 152–167. https://dx.doi.org/10.5614/j.math.fund.sci.2019.51.2.5

[51] Sezgin, A., & Orbay, M. (2022). Analysis of semigroups with soft intersection ideals. *Acta universitatis sapientiae, mathematica*, *14*(1), 166–210. https://doi.org/10.2478/ausm-2022-0012

[52] Atagün, A. O., & Sezgin, A. (2018). A new view to near-ring theory: soft near-rings. *South east asian journal of mathematics & mathematical sciences*, *14*(3). https://rsmams.org/journals/articleinfo.php?articleid=313&tag=seajmams

[53] Manikantan, T., Ramasamy, P., & Sezgin, A. (2023). Soft Quasi-ideals of soft near-rings. *Sigma*, *41*(3), 565–574. https://doi.org/10.14744/sigma.2023.00062

[54] Sezgin, A., Çağman, N., & Çıtak, F. (2019). α-inclusions applied to group theory via soft set and logic. *Communications faculty of sciences university of ankara series a1 mathematics and statistics*, *68*(1), 334–352. https://doi.org/10.31801/cfsuasmas.420457

[55] Muhammad Riaz, M. Ozair. Ahmad, & Khalid Naeem. (2017). Novel concepts of soft sets with applications. *Annals of fuzzy mathematics and informatics*, *13*(2), 239–251. https://doi.org/10.30948/afmi.2017.13.2.239

[56] Sezgin, A., Yavuz, E., & Özlü, Ş. (2024). Insight into soft binary piecewise lambda operation: A new operation for soft sets. *Journal of umm al-qura university for applied sciences*, *1*(1), 79–100. https://doi.org/10.1007/s43994-024-00187-1

[57] Memiş, S. (2022). Another view on picture fuzzy soft sets and their product operations with soft decision-making. *Journal of new theory*, (38), 1–13. https://doi.org/10.53570/jnt.1037280

[58] Sezgin, A., & Ilgin, A. (2024). Soft intersection almost subsemigroups of semigroups. *International journal of mathematics and physics*, *15*(1), 13–20. https://doi.org/10.26577/ijmph.2024v15i1a2

[59] SezgiN, A., Atagün, A. O., & Cagan, N. (2025). A complete study on and-product of soft sets. *Sigma journal of engineering and natural sciences*, *43*(1), 1-14. https://dergipark.org.tr/en/pub/sigma/issue/91176/1661181