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Neutrosophic Sober Open Sets: An Overview

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Abstract

In this study, a new class of neutrosophic sets, known as Neutrosophic sober open sets in Neutrosophic topological spaces, is introduced, and its fundamental features are examined. After this, we create a new topology type, known as a neutrosophic sober topology, in order to study its relationships in more detail.

Keywords: Neutrosophic set, Neutrosophic sober open set, Neutrosophic sober topological space, NŠ(Cl) operator, NŠ(Int) operator.

1 | Introduction

Zadeh [1] introduced fuzzy sets in 1965, which made it possible for things to belong to the unit interval [0,1] to varying degrees of membership. Atanassov [2] expanded this idea in 1983 by proposing Intuitionistic Fuzzy Sets (IFS) [3], which take into account both degrees of membership and non-membership. In 1998, Smarandache [4] presented neutrosophic sets, which were created by adding indeterminacy as a component to the truth and falsity membership functions [5].

Several writers have recently proposed new forms of sets in general topology [6]. A novel class of sets, known as sober open sets in sober topological spaces, was recently introduced by Jayanthi and Jafari [7] in 2024. Additionally, counterexamples have been used to illustrate the sober continuity, sober connectedness, and separation axioms.

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This encourages us to explore the basic features of sober open sets in neutrosophic topological spaces. These concepts have been illustrated with an example. Furthermore, the closure and interior operators of neutrosophic sober topological spaces have been introduced and studied.

2 | Preliminaries

Definition 1 ([4]). Let X be a fixed, non-empty set. A set with the form $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle: x \in X\}$ is called a Neutrosophic set, where $T_N(x)$, $I_N(x)$, $F_N(x)$ represent the degree of truth, degree of indeterminacy and the degree of falsity respectively of each element $x \in X$ to the set N.

Definition 2 ([8]). The complement of a Neutrosophic set N is denoted by N^C and is defined by $N^C = \{\langle x, F_A(x), 1\text{-}I_A(x), T_A(x)\rangle: x \in X\}$.

Definition 3 ([8]). Consider two Neutrosophic sets U and V over X, then U is said to be contained in V, denoted by $U \subseteq V$, if and only if $T_u(x) \leq T_v(x)$, $I_u(x) \leq I_v(x)$, $F_u(x) \geq Fv(x)$.

Definition 4 ([8]). The arbitrary union of two Neutrosophic sets U and V over X is denoted by $U \cup V$ and is defined by $\{\langle x, T_u(x) \vee T_v(x), I_u(x) \vee I_v(x), F_u(x) \wedge Fv(x)\rangle: x \in X\}$.

Definition 5 ([8]). The finite intersection of two Neutrosophic sets U and V over X is denoted by $U \cap V$ and is defined by $\{\langle x, T_u(x) \wedge T_v(x), I_u(x) \wedge I_v(x), F_u(x) \vee Fv(x)\rangle: x \in X\}$.

Definition 6 ([8]). Let N be a Neutrosophic set over X, then the universe set of N is denoted by 1_N and is defined by $1_N = \{\langle x,1,1,0\rangle: x \in X\}$.

Definition 7 ([8]). Let N be a Neutrosophic set over X, then the empty set of N is denoted by 0_N and is defined by $0_N = \{\langle x,0,0,1\rangle: x \in X\}$.

Definition 8 ([8]). Let U and V be two Neutrosophic sets over X, the difference of U and V is denoted by $D_{uv} = U \setminus V$ and is defined by $\{\langle x, T_u(x) \wedge F_v(x), I_u(x) \wedge (1\text{-}I_v(x)), F_u(x) \vee T_v(x))\rangle: x \in X\}$.

Definition 9 ([9]). A Neutrosophic Topology (NT) on a non-empty set X is a family τ_N of neutrosophic subsets in X that satisfies the following axioms [10]:

I. 0N, 1N $\in$ Tn.

II. $U \cap V \in \tau$ for any U, V $\in \tau N$.

III. $\cup Ni \in \tau$ for all $\{ Ni: i \in J\} \subseteq \tau N$.

The pair (X, τN) represents a neutrosophic topological space τN over X.

We denote a neutrosophic open set as an NO set and a neutrosophic topological space as NTS.

3 | Neutrosophic Sober Open Sets

The definitions of a new open set in Neutrosophic topological space, known as a Neutrosophic sober open set and its complement, a Neutrosophic sober closed set, are presented in this section along with a discussion of their fundamental characteristics. To further investigate Neutrosophic sober open sets, a Neutrosophic topological space known as the Neutrosophic sober topological space is also introduced.

Definition 10. A non-empty set $N \neq X$ is said to be a Neutrosophic sober open set in Neutrosophic topological space (X,τN), if there exist two distinct non-empty NO sets $N1 \neq X$ and $N2 \neq X$ in X that satisfy the conditions:

I. $N \cup N1$ is neutrosophic open and $N \cup N1 \neq X$.

II. $N \cap N2$ is neutrosophic open and $N \cap N2 \neq \phi$.

The complement Nc of a neutrosophic sober open set N is a neutrosophic sober closed set in (X,τN). The collection of all neutrosophic sober open (resp. neutrosophic sober closed) sets is denoted by NS-O(X) (resp. NS-C(X)).

Example 1. Let X={a,b} and τN = {0N, A,B,C,D,1N } be a neutrosophic topological space, where

A = ⟨(a,0.3,0.3,0.5), (b,0.4,0.4,0.6)⟩, B = ⟨(a,0.1,0.2,0.3), (b,0.4,0.5,0.6)⟩, C = ⟨(a,0.1,0.2,0.5), (b,0.4,0.4,0.6)⟩ and D = ⟨(a,0.3,0.3,0.3), (b,0.4,0.5,0.6)⟩.

Here, P = ⟨(a,0.3,0.2,0.5),(b,0.4,0.4,0.6)⟩ is an NS-open set since there exist two non-empty NO sets A and B such that P⋃A=A≠ X and P⋂B = C ≠ ϕ.

Q = ⟨(a,0.1,0.3,0.3), (b,0.4,0.5,0.6)⟩ is an NS-open set since there exist two non-empty NO sets D and B such that Q⋃D=D≠ X and Q⋂B = B ≠ ϕ.

R = ⟨(a,0.3,0.3,0.5), (b,0.4,0.5,0.6)⟩ is an NS-open set since there exist two non-empty NO sets D and A such that R⋃D=D≠ X and R⋂A = A ≠ ϕ.

S = ⟨(a,0.2,0.2,0.3), (b,0.4,0.5,0.6)⟩ is an NS-open set since there exist two non-empty NO sets A and C such that S⋃A=D≠ X and S⋂C = C ≠ ϕ.

It is comprehensible that NS-open sets P, Q, R, and S are such that C ⊆ P ⊆ A, B ⊆ Q ⊆ D, A ⊆ R ⊆ D, and C ⊆ S ⊆ D.

Remark 1. A Hasse diagram is one way to represent the NS-open sets in *Example 1* with the relation ⊆.

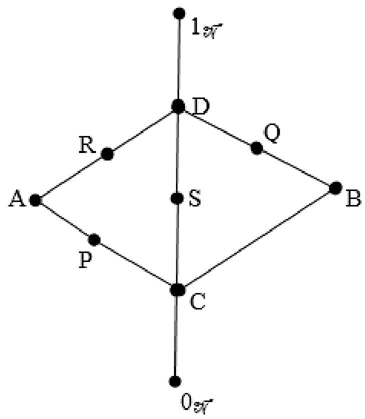

Fig. 1. Hasse diagram of neutrosophic sober open sets with respect to inclusion.

Definition 11. The union of all neutrosophic sober open sets (resp. neutrosophic sober closed sets) in X is denoted by ⋃{NS-O(X)} (resp. ⋃{NS-C(X)}.

Definition 12.

I. The neutrosophic sober complement of NS-O set U is defined by (NS)c(U) = ⋃{NS-O(X)} \ U.

II. The neutrosophic sober complement of NS-C set V is defined by (NS)c(V) = ⋃{NS-C(X)} \ V.

Example 2. This example shows that the neutrosophic sober complement of NS-O set need not be neutrosophic sober closed.

From *Example 1*, the union of all NS-open sets is D.

(NS)c(P) = D \ P = D which is not neutrosophic sober closed.

Remark 2. The neutrosophic sober complement of the NS-C set is always neutrosophic sober open.

From *Example 1*, the union of all NS-closed sets is Cc.

(NS)c(Pc) = Cc \ Pc = P which is neutrosophic sober open.

(NS)c(Qc) = Cc \Qc = Q which is neutrosophic sober open.

(NS)c(Rc) = Cc \ Rc = R which is neutrosophic sober open.

(NS)c(Sc) = Cc \ Sc = S which is neutrosophic sober open.

Example 3. Every NO set in (X, τN) need not be an NS-O set in general.

Let X={u,v,w} and τN = {0N, N, 1N} be a neutrosophic topological space, where N = ⟨(u,0.5,0.6,0.4), (v,0.4,0.5,0.2), (w,0.7,0.6,0.9)⟩.

Here N is a NO set but not an NS-O set, since there does not exist two non-empty disjoint NO sets such that N⋃N1 ≠X and N⋂N2 ≠ ϕ.

Theorem 1. Necessary and sufficient condition for a NO set to be an NS-O set.

A NO set N in (X, τN) is NS-open if and only if there exists a proper NO set M such that N ⊆ M or M ⊆ N.

Proof:

Necessity. Let N be a neutrosophic open set in (X, τN) that is NS-open, then by *Definition 10*, there exist two proper NO sets, say N itself and M, such that N ⊆N ⊆M or M ⊆N ⊆N.

Sufficiency. Assume that N is a NO set in (X, τN) and that if N ⊆ M, where M is a proper NO set in X. Then we have the conditions N⋃N=N ≠X and N⋂M =N ≠ ϕ, both of which are NO in X. If on the other hand, M ⊆N, then we have N⋃N=N ≠X and N ⋂M =M ≠ ϕ, both of which are NO in X. In both cases, N is NS-open in X.

Remark 3. It is obvious that 1N and 0N are not neutrosophic sober open sets.

Theorem 2. A chain has an NS-open set if it contains at least two non-disjoint NO sets.

Proof:

We know that in a chain, every element is comparable. That is, we consider an NTS with two non-disjoint NO sets, say A and B, which can be comparable, then there exists a NS-open set U such that A⊆U⊆B. It is clearly shown in the following figure.

Fig. 2. Existence of a neutrosophic sober open set U in a chain of neutrosophic open sets between two non-disjoint sets A and B.

Remark 4. It is evident that a Neutrosophic topological space (X, τN) with a single NO set does not contain any neutrosophic sober open sets.

4 | Neutrosophic Sober Topological Space

Definition 13. We shall now consider an NTS (X, τN) with at least two non-disjoint NO sets. In this NTS, the following conditions hold:

I. 0N,1N ∈τN.

II. The arbitrary union of two NS-open sets is again NS-open in (X, τN).

III. The finite intersection of two NS-open sets is again NS-open in (X, τN).

We refer to the above NTS as a Neutrosophic sober topological space, and it is denoted as τ_{NSober} and the corresponding space as X_{NSober}.

Definition 14. Let (X_{NSober}, τ_{NSober}) be a Neutrosophic sober topological space. Then the Neutrosophic sober interior and Neutrosophic sober closure of any proper set A in (X_{NSober}, τ_{NSober}) are denoted by NŠ(Int) and NŠ(Cl), respectively, and are defined by

NŠ - Int (A) = ⋃{G : G is NS-open in X_{NSober} and G ⊆ A}.

NŠ- Cl (A) = ⋂{K : K is NS-closed in X_{NSober} and A ⊆ K}.

Remark 5. For any proper set A, NŠ - Int (A) ⊆ A is true only when N Š - Int (A) exists. Also, if A ⊆ B ⇒ NŠ - Int (A) ⊆ NŠ - Int (B) is true for any two proper sets A and B, only when NŠ - Int (A) and N Š - Int (B) exist.

Example 4. Let X={a,b} and τ_{Nsober} = {0_N, A,B,C,D,1_N } be a neutrosophic topological space, where

A = ⟨(a,0.5,0.5,0.3), (b,0.3,0.3,0.2)⟩, B = ⟨(a,0.2,0.2,0.1), (b,0.7,0.7,0.6)⟩, C = ⟨(a,0.2,0.2,0.3), (b,0.3,0.3,0.6)⟩ and D = ⟨(a,0.5,0.5,0.1), (b,0.7,0.7,0.2)⟩.

Here P = ⟨(a,0.4,0.4,0.3),(b,0.3,0.3,0.4)⟩, Q = ⟨(a,0.2,0.2,0.2),(b,0.5,0.5,0.6)⟩, R = ⟨(a,0.3,0.3,0.1),(b,0.7,0.7,0.5)⟩, S = ⟨(a,0.5,0.5,0.2),(b,0.6,0.6,0.2)⟩ and T = ⟨(a,0.4,0.4,0.2),(b,0.5,0.5,0.5)⟩ are some of the NS-open sets.

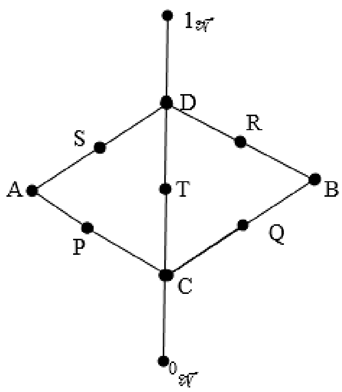

Fig. 3. Hasse diagram representation of the neutrosophic sober topology τNSober on the set X = {a, b}, illustrating the NS-open sets A, B, C, D, P, Q, R, S, and T.

Now we find the NŠ - Int (A) for any proper set of τNsober.

Table 1. Neutrosophic sober interior of selected NS-open sets and proper sets in the neutrosophic sober topological space (X, τNSober).

NS-Open Sets	NŠ – Int (NS-Open Set)	Proper Set	NŠ – Int (Proper Set)
P	P	A	P
Q	Q	B	Q
R	R	C	DOES NOT EXIST
S	S	D	D
T	T		

Theorem 3. Let (XNSober, τNSober) be a Neutrosophic sober topological space. Then for any proper set in XNSober, the following holds:

I. NŠ - Int (A) ⊆ A.

II. A ⊆ B then NŠ - Int (A) ⊆ NŠ - Int (B).

III. NŠ - Int (N Š - Int (A)) = NŠ - Int (A).

IV. NŠ - Int (M ⋂N) = NŠ - Int (M) ⋂ NŠ - Int (N).

V. NŠ - Int (M) ⋃ NŠ - Int (N) ⊆ NŠ - Int (M ⋃N).

Proof: The results are apparent from the above example.

5 | Conclusion

Neutrosophic sober open sets are introduced in this article, along with relevant examples that illustrate their fundamental properties. The concepts of neutrosophic sober interior and closure operators are further discussed in order to support the neutrosophic sober topology. The theorems and properties of neutrosophic sober-open sets in neutrosophic sober topology are explained with related examples. There is also an extent to incorporate continuous functions, connectedness, and compactness based on neutrosophic sober topological spaces.

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Conflict of Interest

The authors declare that they have no conflict of interest.

Data Availability

All data are included in the text.

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